International Journal of Research in Engineering, Science and Management
Volume-3, Issue-7, July-2020
journals.resaim.com/ijresm | ISSN (Online): 2581-5792

On a Semi-Symmetric Metric Connection in Trans-Sasakian Manifold

Savita Verma*
Associate Professor, Department of Mathematics, Pt. L. M. S. Govt. P. G. College, Rishikesh, India
*Corresponding author: savitaverma19sep@gmail.com

***Abstract*: Oubina, J. A. [1] defined and initiated the study of Trans-Sasakian manifolds. Blair [2], Prasad and Ojha [3], Hasan Shahid [4] and some other authors have studied different properties of C-R-Sub –manifolds of Trans-Sasakian manifolds. Golab, S. [5] studied the properties of semi-symmetric and Quarter symmetric connections in Riemannian manifold. Yano, K. [6] has defined contact conformal connection and studied some of its properties in a Sasakian manifold. Mishra and Pandey [7] have studied the properties in Quarter symmetric metric F-connections in an almost Grayan manifold.**

In this paper we have studied the properties of a Trans-Sasakian manifold equipped with a semi-symmetric metric connection.

***Keywords*: Almost-Grayan manifold, C-R-Sub manifolds of Trans-sasakian manifold, Riemannian curvature tensor, Semi-symmetric and quarter symmetric connections in Riemannian manifold, Trans-Sasakian manifold.**

1. Introduction

Let M_n (n = 2m + 1) be an almost contact metric manifold endowed with a (1,1)-type structure tensor F, a contravariant vector field T, a -1 form A associated with T and a metric tensor 'g' satisfying:

(1.1)(a) $F^2X = -X + A(X)T$

(1.1)(b) $FT = 0$

(1.1)(c) $A(FX) = 0$

(1.1)(d) $A(T) = 1$

And

(1.2)(a) $g(\bar{X}, \bar{Y}) = g(X, Y) - A(X)A(Y)$

Where
(1.2)(b) $\bar{X} \stackrel{\text{def}}{=} FX$

And

(1.2)(c) $g(T, X) \stackrel{\text{def}}{=} A(X)$

For all C^{∞}- vector fields X, Y in M_n also, a fundamental 2-form 'F in M_n is defined as
(1.3) $'F(X,Y) = g(\bar{X},Y) = - g(X, \bar{Y}) = -'F(Y,X)$

Then, we call the structure bundle {F,T,A,g}an almost contact-metric structure [1]
An almost contact metric structure is called normal [1], if

(1.4)(a) $(dA)(X,Y)T + N(X,Y) = 0$

Where
(1.4)(b) $(dA)(X,Y) = (D_XA)(Y) - (D_YA)(X)$, D is the Riemannian connection in M_n.
And
(1.5) $N(X, Y) = (D_{\bar{X}} F)(Y) - (D_{\bar{Y}} F)(X) - \overline{(D_X F)(Y)} + \overline{(D_Y F)(X)}$
is Nijenhenus tensor in M_n.
An almost contact metric manifold M_n with structure bundle {F,T,A,g} is called a Trans-Sasakian manifold [3]&[1], if

(1.6) $(D_XF)(Y)=\alpha\{g(X,Y)T - A(Y)X\} + \beta\{'F(X,Y)T - A(Y)\bar{X}\}$

Where, β are non -zero constants.
It can be easily seen that a Trans-Sasakian manifold is normal. In view of (1.6) one can easily obtain in M_n, the relations
(1.7) $N(X, Y) = 2\alpha\, 'F(X,Y)T$

(1.8) $(dA)(X,Y) = -2\alpha\, 'F(X,Y)$

(1.9) $(D_XA)(Y) + (D_YA)(X) = 2\beta\{g(X,Y) - A(Y)A(X)\}$

(1.10) $(D_X\, 'F)(Y,Z) + (D_Y\, 'F)(Z,X) + (D_Z\, 'F)(X,Y)$
$= 2\beta[A(Z)'F(X,Y) + A(X)\, 'F(Y,Z) + A(Y)\, 'F(Z,X)]$

(1.11)(a) $(D_XA)(Y) = -\alpha\, 'F(X,Y) + \beta\{g(X,Y) - A(X)A(Y)\}$

(1.11)(b) $(D_XT) = -\alpha\bar{X} + \beta\{X - A(X)T\}$

Remark (1.1): In the above and in what follows, the letters X,Y,Z …..etc. an C^{∞}- vector fields in M_n.

2. On a Semi-Symmetric Metric Connection in Trans-Sasakian Manifold

We consider a semi-symmetric metric connection B given by [8]

(2.1) $B_X Y = D_X Y + A(X)Y – g(X,Y)T$

Whose torsion tensor is given by

(2.2) $S(X,Y) = A(Y)X – A(X)Y$

The curvature tensor with respect to B, say R(X,Y,Z) is given by

(2.3) $R(X,Y,Z)= B_X B_Y Z – B_Y B_X Z – B_{[X,Y]}Z$

Using (2.1) in it, we get

(2.4) $R(X,Y,Z)=K(X,Y,Z) + (D_X A)(Z)Y - (D_Y A)(Z)X - g(Y,Z)D_X T + g(X,Z)D_Y T +A(Z)A(Y)X - A(Z)A(X)Y +g(X,Z)Y - g(Y,Z)X - A(X)g(Y,Z)T -A(Y)g(X,Z)T$

Again, using (1.11)(b) in (2.4), we obtained

(2.5) $R(X,Y,Z)=K(X,Y,Z)+\alpha\{ 'F(Y,Z)X– 'F(X,Z)Y +g(Y,Z)\bar{X} - g(X,Z)\bar{Y}\}+(2\beta+1)\{g(X,Z)Y -g(Y,Z)X\} -(\beta+1)\{A(Y)g(X,Z)T-A(X)g(Y,Z)T+A(X)A(Z)T-A(Y)A(Z)X\}$

Contracting (2.5) with respect to X, we get

(2.6)(a) $R(Y,Z) = Ric(Y,Z) +\alpha(n-2)\ 'F(Y,Z) –\{(2n-3)\beta + (n-2)\}g(Y,Z) + (\beta+1)(n-2)A(Y)A(Z)$

Or

(2.6)(b) $R(Y) = K(Y) +\alpha(n-2)\bar{Y} + \{(2n-3)\beta + (n-2)\}Y + (\beta+1)(n-2)A(Y)T$

Contracting which with respect to Y, we get

(2.6)(c) $r = k -2\beta(n-1)^2 –(n-1)(n-2)$

Where R(Y,Z), r are Ricci tensor and scalar curvature with respect to B and Ricci and k are respectively the same with respect to Riemannian connection D.

Now, suppose the curvature tensor with respect to B vanishes, i.e. R(X,Y,Z) = 0 then from (2.6)(c), we see that the manifold M_n is of constant scalar curvature k and is given by

(2.7) $\beta = \frac{k}{2(n-1)^2} + \frac{(n-2)}{2(n-1)}$

Also the equation (2.6)(a), in view of the above fact and (2.7) becomes

(2.8) $Ric(Y,Z) = -\alpha(n-2)\ 'F(Y,Z) + \frac{k}{2(n-1)^2}[(2n-3)g(Y,Z) -(n-2)A(Y)A(Z)] + \frac{(n-2)}{2(n-1)}[(4n-5)g(Y,Z) – (3n-4)A(Y)A(Z)]$

Barring Y in (2.8), we have

(2.9)(a) $Ric(\bar{Y},Z) = \alpha(n-2)g(\bar{Y},\bar{Z}) + \frac{(2n-3)}{2(n-1)^2} k\ 'F(Y,Z) + \frac{(n-2)(4n-5)}{2(n-1)}\ 'F(Y,Z)$

Further, barring Z in (2.8), we obtained

(2.9)(b) $Ric(Y,\bar{Z}) = -\alpha(n-2)g(\bar{Y},\bar{Z}) + \frac{(2n-3)}{2(n-1)^2} k\ 'F(Z,Y) + \frac{(n-2)(4n-5)}{2(n-1)}\ 'F(Z,Y)$

Adding (2.9)(a) and (2.9)(b), we get

(2.10) $Ric(\bar{Y},Z) + Ric(Y,\bar{Z}) = 0$

Thus, we have

Theorem (2.1): Let M_n be a Trans-Sasakian manifold admitting a semi –symmetric metric connection B by (2.1)

Let the curvature tensor with respect to B vanish, then M_n is of constant scalar curvature and

$Ric(\bar{Y},Z) + Ric(Y,\bar{Z}) = 0$

Holds good in M_n.

Now, from (2.9)(a) and (2.9)(b), we have

(2.11) $K(\bar{Y}) = \overline{K(Y)} =\alpha(n-2)\{Y-A(Y)T\}+ \frac{(2n-3)}{2(n-1)^2} k\ \bar{Y} + \frac{(n-2)(4n-5)}{2(n-1)}\bar{Y}$

Contracting which with respect to Y, we have

$\alpha(n-1)(n-2) = 0$

which gives $\alpha = 0$,for $n > 2$

then, we have

Theorem (2.2): A Trans-Sasakian manifold M_n, $n\geq 3$, equipped with a semi-symmetric metric connection B given by (2.1) becomes a (0, β) type Trans Sasakian manifold if the curvature tensor with respect to B vanishes.

3. Conclusion

If in a Trans-Sasakian manifold admitting a semi –symmetric metric connection B, and if curvature tensor with respect to B vanish, then M_n is of constant scalar curvature and $Ric(\bar{Y},Z) + Ric(Y,\bar{Z}) = 0$,Holds good in M_n. Again A Trans-Sasakian manifold M_n , $n\geq 3$, equipped with a semi-symmetric metric connection B becomes a (0,β) type Trans Sasakian manifold if the curvature tensor with respect to B vanishes.

References

[1] Obina, J. A.: New classes of almost contact metric structure publ. Math.32(1985), pp 187-193.

[2] Blair, D.E.: Contact manifold in Riemannian geometric lecture note in Math. Vol.509, Springer Verlag, N.4(1978).

[3] Prasad, S. and Ojha, R.H.: C. R. submanifolds of Trans – Sasakian manifold, Indian Journal of pure and Applied Math.24(1993) (7 and 8), pp.427-434.

[4] Hasan Shahid, M" C R sub manifolds of Trans Sasakian manifold, Indian Journal of pure and Applied Math. Vol.22 (1991), pp.1007-1012.

[5] Golab, S.: On semi-symmetric and quarter symmetric linear connections; Tensor, N. S; 29(1975).

[6] Yano, K.: On contact conformal connection; Kodia Math. Rep., 28(1976), pp.90-103.

[7] Mishra, R. S. and Pandey, S.N.: On quarter symmetric metric F-connections; Tensor, N. S. Vol. 31, (1978), pp. 1-7.

[8] Pandey, S. N.: Some contribution to Differential Geometry of differentiable manifolds, Thesis (1979), B. H. U. Varanasi (India).