

Dimensions of Space

Kedar Pansare^{1*}, Meghraj Parab², Vrushabh Parmar³, Yashwantrao Mitnasala⁴, Rajni Bahuguna⁵

^{1,2,3,4}UG Student, Dept. of Civil Engineering, Thakur College of Engineering and Technology, Mumbai, India ⁵Assistant Professor, Department of Humanities and Sciences, Thakur College of Engineering and Technology, Mumbai, India

*Corresponding author: pansarekedar1530@gmail.com

Abstract: The existence and the mysteries of the universe could not be explained by using just 3 spatial dimensions. There was a need to think of higher dimensions as a tool to explain the phenomena happening in our universe. Therefore, unified theories such as Loop Quantum Gravity and Superstring Theory were proposed. We will be taking an overview of these theories in order to get some idea about each.

Keywords: General theory of relativity, Loop quantum gravity, Mobius strip, Special theory of relativity.

1. Introduction

In the early 1900's, Einstein laid the foundation of Modern Physics and introduced us with the concept of Relativity. His gave a new perspective towards space and time. It so happens that the physical rules for the atoms and electrons are totally different than those for macroscopic bodies. This inconsistency of nature encouraged physicists to give a unified theory which could explain all the phenomena in our universe starting from the smallest atomic particle to the largest star. Hence, in late 80's, *Loop Quantum Gravity and Superstring Theory* took different approaches towards solving this problem and used concepts of higher dimensions to give a possible satisfactory solution.

2. Foundation of Today's Physics

In 1686, Newton stated the Law of Universal Gravitation. He told us that all the objects attract each other with a constant force which is proportional to the product of their masses and inversely proportional to the square of the distance between them. This gave a satisfactory explanation to the fact that, "Why every object falls downward and not upward?" It is because of the gravitational force which acts on the object and brings it down to the ground. But one thing that Newton couldn't answer was "What are the causes of gravitational force? Where does it arise from?" Soon after publishing his famous paper on Special Relativity in 1905, this simple question got Einstein thinking, "Can we use Relativity to explain Gravitation?" And the journey of knowing the truth of the Universe had started. [1]

3. Special Theory of Relativity

Our journey begins with the Special Relativity which was

published by Einstein in 1905. The creation of this theory happened due to the following reason: Between 1861 and 1862 James Clerk Maxwell published a paper on Electromagnetism in which he stated certain set of partial differential equations which work in co-ordinance with Lorentz Force Law and today these equations are called as Maxwell's Equations. In these equations, Maxwell stated that light is an electromagnetic wave which propagates in vacuum with speed equal to

$$c = \frac{1}{\sqrt{\mu\varepsilon}} \sim 3 \times 10^8 m/s$$

If Maxwell's laws are valid for all the inertial frames, then the speed of light must be constant in all the Inertial frames. But this doesn't happen in the Classic Newtonian Physics. Suppose we consider two cases:

- 1) A person standing still on his place throws a stone with some velocity *v*.
- A person travelling on a scooter with velocity *u* throws the same stone with same velocity *v*.

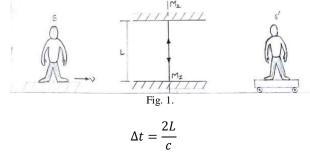
According to Newtonian Physics the stone when thrown while travelling on the scooter should gain more velocity which will be equal to (v + u). But according to Maxwell's laws, this should not be the case with light i.e. when a person travelling on scooter with velocity u fires a laser beam, the velocity of the laser beam will be equal to the velocity of the laser beam which is fired by a person who is standing still on his place. This idea put forth by Maxwell challenged the Newtonian Physics which had given explanation to many of the phenomena happening in nature satisfactorily and were experimentally verified. Something just wasn't right here. Either Maxwell's equations need to be revised or the Newtonian Physics that we have been following since past 300 years needed to be revised. In 1877, an experiment was done called as Michelson and Morley experiment which gave astonishing results. It turned out that Maxwell's equations were correct and proved that light does not need any medium to travel through space and its velocity is always constant. These results from the Michelson and Morley experiment motivated Einstein to formulate a new theory explaining this behavior of light and so he formulated Special Theory of Relativity. There are two main postulates of this theory:



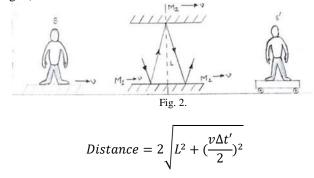
- 1) Speed of light is absolute
- Simultaneity is relativity for events separated by non-zero distance.

A. Light-Beam Clocks and Rods

Consider a hypothetical experiment. We have a rod of length L measured by an observer. Two mirrors are fixed about the two ends of the rod and a light beam is being reflected back and forth between the two mirrors M_1 and M_2 . Let's try to analyze the distance travelled by the light beam in one reflection. Let 'S' be the frame in which a stationary observer is present. When the stationary observer tries to analyze this situation, according to him the light beam will travel distance L, gets reflected by M_1 , and again will travel distance L. From the Fig. 1, the time taken by the light beam for one reflection will be,



Now, let's try to analyze the same situation in another frame 'S' in which the observer is in relative motion with frame S moving with a velocity v towards the left. Since it is a relative motion, it won't be wrong to say that mirrors are in relative motion with the observer moving in right direction with velocity v as well. Here, as the mirrors are moving with velocity v with respect to the observer, the distance travelled by the light beam will change. By using simple geometry, we can determine the distance travelled by the light beam for one reflection (Refer Fig. 2)



Note that the length of the rod has not changed in this frame as well. So, the time taken by the light beam for one reflection will be,

$$\Delta t' = \frac{2}{c} \sqrt{L^2 + (\frac{v\Delta t'}{2})^2}$$

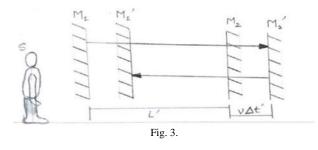
After further manipulations, we finally get,

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t$$
Where, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

One should notice here that, the time taken by the light beam for one reflection when the observer is stationary is not equal to the time taken by the light beam when the observer is travelling with velocity v. This phenomenon is called as *Time Dilation*.

Whenever we are considering cases which deal with matter or objects travelling with a velocity close to that of light, then the proper time interval is always smaller than the improper time interval between the occurrences of two events by the factor γ .

Let's change the scenario a bit. Now consider the light beam clock is moving with a velocity v parallel to its length. In this case we are not sure whether the length of the rod remains constant or not. Let us consider that the length of the rod in frame S is L'. The light pulse is moving towards M₂ after getting reflected from M₁. But M₂ itself is moving with a velocity v. Consider that the light beam stuck the mirror M₂ when it moves to a position M2' and takes time $\Delta t_1'$ to get reflected back and reach M₁when it reaches position M₁'. (Refer Fig. 3)



The mirror M2 has moved to M2' in time $\Delta t_1'$. The distance travelled by it will be $v\Delta t_1'$. This implies that the light beam has travelled $L' + v\Delta t_1'$ before striking M₂. Speed of light is c and it must have covered a distance of $c\Delta t_1'$ in time $\Delta t_1'$. Hence,

 $c\Delta t_1' = L' + v\Delta t_1'$

Which implies,

$$\Delta t_1' = \frac{L'}{c - v}$$

In the same way, time taken by the light beam to travel back from M_2 ' to M_1 ' will be,

$$\Delta t_2' = \frac{L'}{c+v}$$

The total time taken by the light beam in its journey is,

$$\Delta t = \Delta t_1' + \Delta t_2' = \frac{L'}{c - v} + \frac{L'}{c + v} = \frac{2cL'}{c^2 - v^2}$$

A stationary observer will observe that the time interval is



 $\Delta t = \frac{2L}{c}$. This is the proper time whereas, $\Delta t'$ is improper time interval as it measures time for events that occur at different place.

$$\Delta t' = \gamma \Delta t = \gamma \left(\frac{2L}{c}\right)$$
$$\frac{2L'c}{c^2 - v^2} = \frac{2L}{c\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\frac{L'}{1 - \frac{v^2}{c^2}} = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Therefore,

$$L' = L \sqrt{1 - \frac{v^2}{c^2}} = L\gamma$$

This equation suggests that the length of rod has contracted by the factor γ if it moves parallel to its length. The proper length is always greater than the improper length of the rod. This phenomenon is called as *Length Contraction*.

From the above discussion one can conclude that the speed of light is absolute even when observed from different frames at different velocities. If one has to agree on the fact that the speed of light is absolute, then he has to disagree on time and space being absolute (i.e. factors of speed of light). While travelling at a speed closer to the speed of light, one can observe length contraction and time dilation at a dramatically higher rate. The reason that we do not observe these effects in our day to day life is because we travel with a very negligible velocity as compared to that of light. These phenomena are observed only the speed is comparable to that of light. [2]-[5]

B. Speed of Light – The Ultimate Speed?

It is a very common result in *Special Theory of Relativity* that no particle than exceed the speed of light. This implies that no information can be transferred with a speed greater than *c*. But in 1962, O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan suggested that there may exist particles which travel with a speed greater than *c*. These hypothetical particles are called as *Tachyons*. However, it turns out that particles that always travel with speed greater than that of light violet the laws of physics that we know. According to *Special Theory of Relativity* it will lead to the violation of causality. These means that in a world where all the particles travel with speed greater than *c*, the effect will precede its cause. E.g. A bullet will hit the object even before it is fired. [6], [7].

4. General Theory of Relativity - Time as 4th Dimension

These ideas that Einstein gave changed the whole world and its perspective towards time. In 1916, Einstein gave a modified theory which was the extension of *Special Theory of Relativity*. Einstein mathematically suggested that Gravitation is a geometric property of space and time. Space and time are very closely related. One might imagine space and time to be different entities. But Einstein proved that space and time are interdependent and should be thought as one entity. They are inter-woven creating the fabric of space-time. When a body of a mass is introduced in this space, because of its energymomentum, it bends the space-time curvature. This is the cause of the attractive force between the object and other matter present in the universe. This theory of Einstein treated time as the 4th dimension.

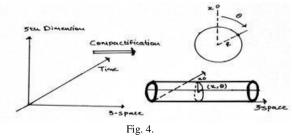
General Theory of Relativity added a new dimension, the 4th dimension which was Time. We experience 3 dimensions (i.e. length, breadth and height) in our day-to-day lives. But time as a dimension is hard to imagine which Einstein has simplified for us. According to Einstein, time is a dimension. If so, can we travel in time? Does time travel possible? To answer this question, we first need to decide the direction in which one must travel. If we consider that one must travel in future, then the answer is: Yes! We can. According to General Relativity, time passes slowly when one experiences gravity. The rate of slowing of time depends on the strength of the gravity which is the result of bending of spacetime curvature. Consider this example; You are in a spaceship which is being controlled from Earth. You encounter a Black Hole. Now, black holes have the mass equivalent to that of 4 million suns which is crushed down to a single point because of its strong gravitational force. This means that the time will drastically slow down for you. This makes it a natural time machine. The average radius of a black hole is 50 million miles. Let's imagine that your spaceship somehow avoids getting sucked inside the black hole and start revolving around it which is of diameter 30 million miles. According to the clocks on earth, every revolution would take 60 mins but the clocks on the ship will be slowed down and will be able to experience just the half of time. For every 60 minutes orbit you would only experience 8 minutes. Imagine you obit the black hole for 5 years, and when you return to Earth everyone has aged by 10 years. You are a time traveler. You not only have made a journey in space but also in time.

Let us think about this question "Can we travel in the past?" One of the interesting things about the equations that Einstein gave is that they don't suggest that time travels in one direction. The laws of physics are applicable when time travels forward as well as backward. But it is in contradiction to our daily experience. Here, we need to consider a concept called 'The Arrow of Time' which was given by Arthur Eddington in 1927. This concept is based on the Second law of Thermodynamics which states that the total entropy of a system can never decreases over time. This means that a building will decay over time and will disintegrate. There will never be a case where the building's disorder decreases over time. Similarly, time should also travel in only one direction. But this is one of the unanswered questions in physics. We don't know whether it is possible to travel in the past. But according to Einstein, as we can know that space is out there, we can state that time is also out there. The past, the present and the future exists. Time is just an illusion.



5. Way Towards the Higher Dimension

After reviewing Einstein's General Relativity, Theodor Kaluza stated a Unified theory in which he tried to apply the same equations to the electromagnetic force as that stated by Einstein for gravitational force. Since Einstein had stated that gravitational force works in the 4 dimensions, the idea was that if we want to include one more force then we need to add one more dimension. When Kaluza derived the equation for electromagnetic force in fourth dimension, he found that it was the equation that we had long known as electromagnetic equation. There was an addition of one more dimension – The 5th Dimension. The obvious questions arise "If there are five dimensions than why can't we see the 5th dimension?", and "Does this theory actually apply to the real world?" In 1926, Oskar Klein suggested that there might be two types of dimensions, ones that are large enough to be seen by us and the others which are so small that they are curled up inside the space and so remain unnoticed by us. E.g. A cable when observed from far away distance, it seems like a straight line. But for an ant crawling on the same cable, there is a diameter to the cable, and it is a 3-dimensional cable.



As shown in the Fig. 4, if we just observe the space itself at a very microscopic level, we might see this 5th dimension in circles of radius 10^{-30} cm. Therefore, 5th dimension is a compact space and is called as *Compact Dimension*. [8], [9]

Answering to the 2nd question, "Does this theory apply to the real world?" Along with Einstein and Kaluza, many others tried to prove this theory, but the theory couldn't be proven and hence was eventually discarded.

6. Loops and String

Currently, there are two theories that are trying to unify Einstein's General Relativity and the electromagnetic force which are:

- 1) Loop Quantum Gravity
- 2) Superstring Theory

A. Loop Quantum Gravity

Loop Quantum Gravity (LQG) is an ideology of Quantum Gravity that includes quantum mechanics and Einstein theory of Relativity. It is a vigorous attempt to establish a theory which includes gravity and other three fundamental forces. It is a leading challenger to String theory.

Loop Quantum Gravity starts with general relativity and then quantum features are added. According to Einstein, "Gravity is not a force, it is the property of space -time itself'. Based on Einstein geometrical formulation, *Loop Quantum Gravity* is an effort to create a *Quantum Theory of Gravity*.

The way energy and momentum are quantized in the quantum mechanics, the same way space and time are quantized in *Loop Quantum Gravity*. This theory also explains that space-time is 'granular'. This means that space-time exists in packets of Planck cubes with a volume of Lp3. This theory explains the resemblance of space and time. Just like separate energy levels, space here itself is discrete. And there is a minimum distance to pass through it. Space can be seen as a finite loop of very thin fabric or fine networks where network of loops is called as spin foam. The size of the structure is about 10⁻³⁵m. [10-12]

Thus, *Loop Quantum Gravity* estimates that space itself has an atomic structure just like matter.

Physical Applications of LQG [13]:

- 1) Black hole entropy
- 2) Planck star
- 3) Hawking radiations in LQG
- 4) Loop quantum cosmology

B. Superstring Theory

String Theory is an attempted to explain each and everything that is present in the universe from matter to different kind of forces and how they work. The Superstring Theory has multiple versions, but they are highly related to each other. String Theory could be referred to as the theory of everything, explaining how this universe is formed. The basic idea is having a unified theory which could explain every phenomenon occurring in the universe. Superstring Theory states that the most fundamental part of matter consist of vibrating filament of energy strings. As it suggests that it is the most fundamental particle of atom (nucleus) and similar vibrating particle makes up an atom. All the forces, matter, etc. are made of these vibrating energy strings. This theory suggests that, in space, there is not a just a single universe but there are multiple universes. We are living in one of the universes present in this multiverse.

According to Einstein, our universe is like an expanding soap bubble and we are at its surface. But this theory suggests that there are multiple soap bubbles just like our universe which are out there and when these bubbles of universes collide or spilt into half, the phenomenon that takes place is none other than the big bang. *Superstring Theory* has also given a satisfactory explained of the big bang. But when we try to work out this theory mathematically, it turns out that it is not applicable to a world which has 4 dimensions nor 5 but to a world which has a total of 10 dimensions- 9 dimensions of space and one dimension of time. [1], [14]-[17]

7.10 Dimensions

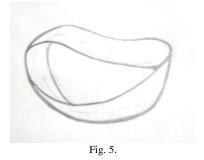
Let's start with the 0th dimension. A point in space is the 0th dimension. It doesn't have any length or height. It is just a point. We draw one more such point in space and connect them with



a line. This a 1-dimensional line. It has a length. Draw one more such line in space and connect them, we get our 2nd dimensional object which has a length and a breadth. Consider a 2-dimensional square. We draw one more square above it and join their edges with lines. We get a cube which is a 3dimensional object having length, breadth and height. We experience these 3 dimensions in our daily life. Are you able to see what are we doing with these dimensions? We are stacking one dimension above the other. The question naturally arises, "How would it feel to be in the 4th dimension? Einstein said that time is the 4th dimension. How would a line in 4th dimension look like?" If we consider time as past, present and future, then a line in 4th dimension would be a line joining the past and the future. If you investigate the 4th dimension, you would see a long undulating snake with your embryonic self at one end and your deceased self at the other. While viewing the bigger picture, you will be able to see a line starting from the big bang till the possible endings of our universe. But since we are trapped inside the 3-dimensional world, we are unable to see this timeline.

Interesting thing about stacking one dimension above other is that the actions that we take in the lower dimensions cause a dramatic effect in the higher dimension and yet we remain unaware about it. Consider the example of a *Mobius strip*:

One of the most famous surfaces is the *Mobius strip*. It can be made by cutting an ordinary strip of a paper. Produce a half twist in it and adhere the ends of the strips together. From the above construction, it can be observed that the *Mobius strip* has a single side only whereas the original paper clearly had a pair of sides. You can even check this out by drawing a line with marker on both the sides of the strip and compare the original strip with the *Mobius strip*. (Refer Fig. 5) [18]



Let us imagine you are a 1-dimensional traveller, travelling along a line on the *Mobius strip*. You start at some point to travel in one direction along the line. After completing one revolution, you should eventually find yourself where you started, Right? But the answer is No. Instead you will find yourself upside-down. It has a mathematical property of being non-orientable. Since, you- the 1-dimensional traveller are travelling along the *Mobius strip*, the 3-dimensional strip is making twists and turn, and you remain unaware about it and find yourself upside-down. There are various applications of *Mobius strip* such as:

- 1) Mobius aromaticity
- 2) Mobius resonators
- 3) Mobius resistors [19]

Just like the *Mobius strip*, we humans trapped inside the 3dimensional world remain unaware of the twists and turns taken by the higher dimensions and just experience the result of it. The line in the 4th dimension might seem to be a straight line but is actually twisting and turning in the 5th dimension. This 4-dimensional timeline is changing continuously in the 5th dimension. The long undulating snake of our life which seemed like a straight line in the 4th dimension will now have branches in the 5th dimension which are influenced by our life choices, chances and actions of others.

As we are going higher and higher in dimensions, we are getting a clearer picture of each timeline which is possible in this universe having equal possibility. The future changes according to your life choices, chances and actions of others. Quantum Mechanics suggests that the sub-atomic particles vibrating along an axis, collapse from the wave of probability simply by the act of observation.

What if you were to see yourself as the richest man in the world in the respective possible timeline? To do so you go up to the 4th dimension and jump into your past to visit your younger self through the 5th dimension. To see yourself as the richest man you'll have to trigger the events that could make it happen. For e.g. you invent computers to get rich. But in this timeline also, you won't be able to get to the point where you become rich because the timeline has not yet unfolded in the 5th dimension. For that you'll have to wait for the timeline to unfold. This will be a long cut. Instead you can jump through the 6th dimension by folding the 5th dimensional branches (like the *Mobius strip*) and visit your future.

Things get even more interesting as we go higher. To define the 7th dimension, we'll have to treat the 6th dimension as a single point. To do so, we'll have to take into consideration all the endless possibilities from our universe coming into existence because of the big bang till the possible endings of our universe. This for us is infinity! In the 7th dimension we'll have to treat this infinity of infinite timelines which could have or will have taken place in the 6th dimension as a single point. When we consider this single point in the 7th dimension, we are just imagining all the possibilities of our universe. To draw a line in the 7th dimension we'll have to draw one more point just like this one because our line will be joining these two points. For that we'll have to define another point as we already have considered all the infinite possibilities of our universe. To draw another point, we'll have to consider another infinity which was caused by a different universe having different initial conditions which are different from our own big bang. Different initial conditions will create different universes and so will create different infinities. So, the line joining any two infinities will be a line in the 8th dimension. Just like the Mobius strip, we can jump from one infinity to the other in the 9th dimension. If one has to travel to a different universe, he will have to jump



through the 9th dimension. As there could be infinitely many universes to branch out and jump into, we'll have to imagine all these infinities as a single point in the 10th dimension. Now if we consider drawing a line in the 10th dimension, we'll need another point. But there's no other place left to go. Since we have considered all the possibilities, our journey is over.

8. Conclusion

It was Einstein's work which gave a satisfactory explanation about the true nature of gravity and time. His Theory of Relativity declared Time as a new dimension which was responsible for creating foundation of higher dimension. In order to propose a unified theory which is capable of explaining all the phenomena happening around us and, in this universe, right from Quantum Mechanics to gravity, various theories have come up. The leading two theories in the world which are trying to do so are *Loop Quantum Gravity* and *Superstring Theory*. Both the theories believe, in their own ways, that the world has higher dimensions which we are unable to see. Even though these two theories are yet to be proven, but still we are sure that they have changed our perspective of approaching time and our universe. The universe is yet to unfold.

Acknowledgement

Preparing a review paper on such a vast topic was a very difficult task for us. It would not have been possible to prepare such a paper without the help of others. Hence, we would like to thank all our friends, and teachers for helping us in this technical paper. We are extremely thankful and pay our gratitude to our faculty Dr. Sandhya Maheshwari for her support and guidance. They have encouraged us, and it was because of them that we have successfully prepared this review paper. We would also like to thank H&S Department, TCET for giving us this opportunity. Thank you.

References

- [1] B. Greene, The Elegant Universe: Superstring, Hidden dimensions, and the Quest for the Ultimate Theory, 2011.
- [2] H. C. Verma, Concepts of Physics 2, volume 2, 2013.
- [3] R. R. Yadav, D. Singh, S. P. Singh, and D. K. Pandey, Modern Physics for Scientists and Engineers, 30 September 2013.
- [4] G. Aruldhas, P. Rajagopal, Modern Physics, sixth printing, July 2014.
- [5] K. Sivaprasath, Modern Physics, 31 December 2008.
- [6] L. Randall, Warped Passages: Unraveling the Mysteries of the Universe's Hidden Dimensions, 3 August 2006.
- [7] Tipler, Paul A., Llewellyn, Ralph A., Modern Physics, 5th edition, 2008
- [8] Klein, Oskar, The Atomicity of Electricity as a Quantum Theory Law, 1926.
- [9] Goenner. H, "Some remarks on the genesis of scalar-tensor theories", General Relativity and Gravitation, 2012.
- [10] C. Rovelli, "Loop Quantum Gravity", August 2008, retrieved 14 September 2014.
- [11] S. Mercuri, "Introduction to Loop Quantum Gravity", 5th International School on Field Theory and Gravitation, April 20-24, 2009.
- [12] C. Rovelli, Physics World, volume 16, number 11, November 2003.
- [13] C. Rovelli, "Loop Quantum Gravity", Living Rev Relativity, 11 (2008).
- [14] J. H. Schwarz, "Physical states and pomeron poles in the dual pion model", Nuclear Physics, 1972.
- [15] H. Sahlmann, "Loop Quantum Gravity", Foundation of Space and Time: Reflection on Quantum Gravity, 2012.
- [16] S. Hossenfelder, "String Theory Meets Loop Quantum Gravity", Quanta magazine, 12 January 2016.
- [17] B. Greene, "String Theory on Calabi-Yau manifolds", Lect at TASI-96 Summer School, Fields, Strings and Duality.
- [18] C. A. Pickover, The Möbius Strip: Dr. August Möbius's Marvelous Band in Mathematics, Games, Literature, Art, Technology, and Cosmology, March 2005.
- [19] Davis, Richard L., "Non-inductive electrical resistor", 1966.