International Journal of Research in Engineering, Science and Management
Volume 8, Issue 11, November 2025
www.ijresm.com | E-ISSN: 2581-5792 | RESAIM Publishing (www.resaim.com)

Optimization of Air Velocity and Fan Speed for Improved Outlet Temperature in Water–Air Heat Exchanger Drying Systems

Kenneth Korkoren[1], Titus Rotich[2*], Samwel Rotich[3]

[1]*Student, Department of Mathematics, Physics and Computing, Moi University, Eldoret, Kenya*
[2,3]*Supervisor, Department of Mathematics, Physics and Computing, Moi University, Eldoret, Kenya*

***Abstract*: The increasing demand for sustainable thermal energy solutions in food processing. House heating and other domestic applications has intensified interest in solar-assisted drying systems, particularly those integrating heat exchangers to deliver controlled drying air conditions. Efficient dehydration of agricultural products requires precise regulation of air temperature, humidity, and flow rate to preserve quality while minimizing energy consumption. However, determining the optimal combinations of these parameters remains a major challenge, especially for systems relying on variable renewable energy sources such as solar thermal input. This study develops a comprehensive mathematical model for a solar-powered food dryer incorporating a counter-flow water-to-air heat exchanger and a forced-convection drying chamber. The model describes heat and mass transfer dynamics using thermodynamic laws, fluid flow relationships, and heat exchanger effectiveness–NTU formulations. A detailed derivation of heat exchanger performance is presented, linking outlet air temperature to key variables including water inlet temperature, heat transfer area, overall heat transfer coefficient, air velocity, fan sweep area, and volumetric flow rate. By reformulating the number of transfer units (NTU) as a direct function of air velocity, an explicit expression for air-side control of outlet temperature is obtained. Simulations using experimentally realistic parameters show that air velocity, fan speed, and fan diameter exhibit inverse relationships with outlet air temperature; reducing air mass flow increases residence time within the exchanger, thereby improving thermal pickup and raising outlet temperatures. The drying chamber temperature evolution, and air–product interaction is regulated using a feedback-based control strategy to obtain robust drying conditions throughout the dehydration process, ensuring that food products are exposed to temperatures below their thermal degradation thresholds. Pontryagin's Maximum Principle was used to determine the optimal drying air temperature, with respect to airflow velocity, and fan speed. Results demonstrate that a heat exchanger of area 2.827m^2 with inlet water temperature at 00^{0}C flowing at $V_l = 1.128l/s$, can transfer heat up to 75.6^{0}C of air temperature up from 25^{0}C working at efficiency of $\epsilon(v) = 0.6826$ and fitted with a fan of are $A_f = 0.05m^2$ spinning at $v = 20RPS$. These parameters can be adjusted for a different desired output air temperature. The optimization model framework offers a tool for enhancing dryer performance, guiding design decisions, and reducing energy costs in solar-assisted food processing systems.**

***Keywords*: Heat Exchanger, Air mass flow, Optimization, Controls, Fan speed, Dryer.**

1. Introduction

Heat also called thermal energy, is a type of energy existing in substances or materials due to vibration of particles, which creates temperature differences. Heat energy is transferred from points of high temperature to points of lower temperature, either by conduction (direct contact of good conductors of heat), by convection (through the fluid as a result of movement of particles), or by radiation (transfer by electromagnetic waves with no media) [1].

Heat energy, (measured in Joules (J), kilowatt - hour (kWh) or Calories (Cal) depending on the context), is very essential and in everyday life, as it is used for various purposes, which include; domestic use for heating spaces, cooking food, industrial use; manufacturing, chemical processing, drying and dehydration, electric generation, thermal imaging, plant growth, laboratory experiments, steam engines and other mechanical applications [2].

Solar radiation constitutes the fundamental and most abundant source of thermal energy available to the Earth. As a naturally occurring and renewable resource, solar energy is freely accessible at the point of incidence. However, the economic costs associated with its utilization arise primarily from the infrastructure required for its collection, conversion, and distribution to end-use applications. Depending on the specific energy demands, solar radiation can be transformed into various usable forms, including thermal, electrical, and mechanical energy, through appropriate technological pathways.

Astrophysically, the Sun is a massive, self-gravitating sphere composed predominantly of gaseous elements under extreme pressure and temperature conditions. Its energy output is sustained by nuclear fusion reactions occurring in its core, wherein hydrogen nuclei undergo fusion to form helium, releasing vast quantities of energy in the form of heat and electromagnetic radiation. The Sun's elemental composition is approximately 70% hydrogen, 28% helium, 1.5% carbon and

*Corresponding author: drtkchebion@gmail.com

oxygen, and 0.5% other trace gases. These constituents are gravitationally bound, maintaining the Sun's spherical geometry and enabling continuous radiative output that drives terrestrial energy systems. Many other elements like neon, iron, silicon, magnesium, sulphur are found in small quantities in the sun [3].

The Sun's surface temperature is approximately 5,700 °C, and its radiative output reaches the Earth through the process of insolation. This incident solar radiation serves as the primary driver of terrestrial thermal dynamics, heating the Earth's surface to varying degrees depending on several factors, including surface material properties, topographical elevation, and geographic location. Consequently, surface temperatures exhibit significant spatial variability. Empirical records indicate peak surface temperatures of approximately 58 °C in Africa, 54 °C in Asia, and 50 °C in Europe, with extreme lows reaching 15 °C in Antarctica.

Harnessing this solar radiation for domestic and industrial applications is achievable through photovoltaic (PV) technology. Solar PV cells convert incident radiation into electrical energy, which can be deployed for a range of end-use functions. These include thermal applications such as water heating, space heating in residential and commercial buildings, operation of drying chambers for agricultural or food processing purposes, and powering of electrical machinery. The integration of solar energy systems into domestic infrastructure represents a viable pathway toward sustainable energy utilization, particularly in regions with high solar irradiance [4].

Solar thermal systems convert incident solar radiation into usable thermal energy through collectors such as flat-plate panels or evacuated tubes. In domestic settings, they are widely employed for water heating, space heating, and operating food drying chambers. Industrial applications include process heat for sectors like chemical manufacturing, petroleum refining, and food processing, where high-temperature steam is required. These systems reduce dependence on fossil fuels and mitigate greenhouse gas emissions. Solar thermal technologies can replace fossil-based heat in industries responsible for over 15% of total CO_2 emissions [5]. Their integration into residential and commercial infrastructure supports energy independence and cost savings.

Apart from solar thermal energy, geothermal energy is another source of natural thermal energy derived from the Earth's subsurface heat, accessed via wells or heat exchangers. It is categorized into direct-use systems (for heating buildings, greenhouses, and spas) and indirect-use systems (for electricity generation via steam turbines). Ground-source heat pumps are commonly used for residential heating and cooling, offering high efficiency and low environmental impact [6]. Globally, geothermal power capacity rose from 15 GW in 2020 to approximately 18 GW in 2023, with projections exceeding 25 GW by 2030. Its applications span district heating, industrial drying, and base-load electricity generation.

Thermal energy is quantitatively assessed through the principles of calorimetry and thermodynamics. The most fundamental expression for calculating the amount of heat energy transferred to or from a substance undergoing a temperature is given by the equation,

$$Q = m.c_p \Delta T \tag{1}$$

where Q denotes the measure of the amount of energy, m denotes the mass of the material, c_p is the specific heat capacity of the material, and ΔT is the change in temperature.

This equation assumes no phase change and constant pressure conditions. Experimental determination of thermal energy typically involves calorimetric techniques, wherein the temperature change of a known mass of fluid (often water) is measured following energy exchange [7]. These formulations underpin thermal analysis in fields ranging from food engineering, house heating, greenhouse heating, and other systems.

2. Thermal Energy and Food Drying Technologies

Thermal energy from the sun can be trapped and utilized locally in various areas of applications. The most common application is solar heating and solar electric power. These two applications both use a special solar panel which can transform the energy from the sun to other forms. Several studies have been done on the improvement of efficiency, and application of thermal energy in various fields. Most of the studies use mathematical equations to model thermal heat dynamics.

The solar domestic hot water model studied composed of the solar collector, storage tank and the greenhouse, all interconnected with a series of insulated pipes ferrying hot and cold water. Heat and mass transfer equations were used in modeling the heat dynamics in a greenhouse. Block oriented approach was used in modeling domestic solar heating. Simulation were run for a period of 24-hours using MATLAB-SIMULINK, and the model results showed significant equivalence with the measured data [8].

As an intervention strategy to food security, studies on solar food drying technology was done by [9]. The study focused on a fully packed continuous conveyor direct fired dryer. Food and drying air mass and energy balance equations were used to model the dryer. Simulation results were run to optimize the drying air velocity and temperature parameters. In this study, methane fuel was used as a source of energy, which hated hot air circulation in the dryer. It was found that mass and energy transfer equations were able to model the drying process of the food particle. Model optimal operation was found to closely equivalent to the experimentally measured data. In this model, the dryer is a directly heated continuous conveyor belt, with direct heating from burning methane gas.

Drying technologies of food was also studied by [10]-[12]. In their study, various characteristics of food are discussed. These include physical and chemical changes experienced during drying. The effect of moisture popularly known as water activity is explored in detail. Various drying technologies and dehydration processes are discussed. The various modes of heating explored include; convectional heating, radiative heating, microwave. Radio frequency and joule heating are explored.

A mathematical model of the drying process of skin and hides was studied by [13]. Two fundamental equations on heat and mass transfer were used to model the drying process. It was found that transient distribution of moisture and heat in leather can be predicted and therefore optimized to conserve energy.

Studies on drying process of vegetables was done by [14] using a rotary dryer. Air and product temperature and humidity was determined and compared with actual data measurement and significant correlation was found. Six differential equations were analyzed to solve and determine the optimal ranges of six parameters, which include inlet temperature, air flow rate, moisture loss rate, outlet air temperature and humidity. From dynamic simulation, it was shown the temperature and moisture content of the products and the drying air can be predicted and therefore determine the drying time required for each product. Inlet air temperature was found to be more significant in the drying process than the air flow rate in a parallel flow, but for concurrent flow, the air flow rate was found to be significant component in the drying process [14].

The big question which this research study is trying to answer is, for how long, at what temperature, at what air flow rate and velocity, and what initial moisture content of a product is required to dehydrate high quality final product to desired moisture content, and at what cost of energy. Is the cost minimal? These are answered from the model to be formulated, and simulations to be run.

A. Optimization of Food Drying Process

The optimization of the drying process is crucial for maximizing efficiency, preserving the quality of the dried products, and minimizing energy consumption. Several factors can be considered to optimize the drying process, ensuring consistent and high-quality results. First, controlling the drying temperature and airflow is essential. The temperature should be carefully regulated to ensure efficient moisture removal without causing thermal degradation or loss of nutritional value in the food product. Additionally, optimizing the airflow within the drying chamber is critical to facilitate moisture evaporation and maintain uniform drying throughout the product. The proper selection of the drying method and equipment is also important for optimization. Different foods may respond best to specific drying techniques, such as air drying, oven drying, freeze-drying, or use of commercially available food dehydrators. Each method has distinct advantages and limitations, and choosing the most suitable approach for the specific food product is essential for achieving optimal results. Furthermore, the thickness and size of the food pieces being dried should be carefully considered. Uniformly sizing the food items and maintaining a consistent thickness can help facilitate uniform drying and reduce the overall drying time, leading to increased efficiency and consistent product quality. Monitoring and controlling the relative humidity in the drying environment are critical for optimizing the process. Maintaining proper humidity levels helps prevent case hardening, where the surface of the food dries too quickly, forming a barrier that impedes moisture evaporation from the interior of the product. This can be achieved through the use of appropriate ventilation, air circulation systems, and humidity control measures. In addition, the prevention of microbial contamination and spoilage during the drying process is essential. Implementing proper hygiene practices, sanitation, and preventive measures during pre-drying and post-drying stages can help ensure the safety and quality of the dried food products. Lastly, optimizing the overall energy efficiency of the drying process is crucial for sustainable and cost-effective operations. This may involve the use of energy-efficient drying equipment, integration of renewable energy sources, and implementation of heat recovery systems to minimize energy consumption and operational costs. Careful attention to factors such as temperature control, airflow, equipment selection, food piece size, humidity management, hygiene practices, and energy efficiency is therefore necessary in the optimization process. By considering these factors and implementing best practices, it is possible to achieve optimal drying results, maintain product quality, and ensure the safety and efficiency of the drying operations.

As much as the target in food drying process is to achieve a threshold value of moisture content, that inhibits decay of food products, the process of doing it should be optimal in terms of time, cost and quality control. In order to achieve these, various parameters need to be varied and evaluated through simulation, to determine the optimal values. One of the major concerns is environmental conservation and use of green energy. The most common sources of heat in food dryers include electricity, fuel and thermal energy commonly referred to as energy.

B. Mathematical Optimization Techniques

Mathematical optimization methodologies encompass a diverse set of techniques and algorithms designed to identify the best solution from a range of feasible options, subject to specific constraints and objectives. These methodologies are utilized across various fields, including operations research, engineering, economics, finance, and machine learning, to address complex decision-making problems and enhance efficiency and performance. One prevalent optimization methodology is linear programming, which is employed to address problems characterized by linear relationships and constraints. Linear programming aims to optimize a linear objective function subject to a set of linear equality and inequality constraints. By utilizing techniques such as the simplex method and interior-point methods, linear programming facilitates the allocation of resources, production planning, supply chain optimization, and other decision-making processes in diverse applications. Another prominent optimization technique is nonlinear programming, which addresses problems with nonlinear objective functions and constraints. Nonlinear programming involves the use of optimization algorithms such as gradient-based methods, genetic algorithms, and simulated annealing to find optimal solutions in cases where relationships between variables are nonlinear. Nonlinear programming is instrumental in optimizing complex systems, including engineering designs, financial modeling, and parameter estimation in machine learning. Integer programming and mixed-integer programming methodologies are employed to tackle problems where decision variables are required to take integer values, adding an additional layer of complexity compared to linear programming. These methodologies are crucial in addressing optimization challenges related to discrete decision-making, such as resource allocation, scheduling, and combinatorial optimization problems in logistics, transportation, and telecommunications. Furthermore, optimization methodologies extend to stochastic programming, which accounts for uncertainty and risk in decision-making processes. Stochastic programming models incorporate probabilistic distributions and scenarios to address optimization problems under uncertainty, making them essential in risk management, portfolio optimization, and decision-making in the presence of random variables and fluctuations. Additionally, metaheuristic optimization techniques, including genetic algorithms, particle swarm optimization, and simulated annealing, offer efficient approaches to solving complex optimization problems by leveraging population-based search strategies and iterative improvement mechanisms. These methodologies are particularly effective in addressing combinatorial optimization, global optimization, and multi-objective optimization challenges in diverse domains. Moreover, convex optimization methodologies are pivotal in addressing optimization problems characterized by convex objective functions and constraints. Convex optimization techniques, such as interior-point methods and gradient descent algorithms, enable efficient and scalable solutions for a wide range of applications, including machine learning, signal processing, and control systems. In conclusion, mathematical optimization methodologies encompass a rich array of techniques and algorithms that play a critical role in addressing complex decision-making problems across diverse domains. By leveraging linear programming, nonlinear programming, integer programming, stochastic programming, metaheuristic optimization, and convex optimization methodologies, researchers, engineers, and decision-makers can derive optimal solutions, enhance efficiency, and make informed decisions that drive progress and innovation in a wide range of industries and applications.

C. Feedback Mechanism and Optimal Controls

Feedback mechanisms are a fundamental concept in various fields, including biology, engineering, business, and social sciences. A feedback mechanism involves the process of using information about the output of a system to influence the operation of the system itself. This loop of information allows a system to adjust its behavior based on the output, thereby regulating its performance, stability, and response to external stimuli.

In biological systems, feedback mechanisms play a critical role in homeostasis, which is the maintenance of internal stability and balance. For example, in the human body, the endocrine system uses feedback mechanisms to regulate hormone levels, blood sugar, body temperature, and other physiological variables. Negative feedback loops help maintain stability by counteracting deviations from a set point, while positive feedback loops can amplify changes and lead to system instability [15].

In engineering and control systems, feedback mechanisms are essential for regulating the behavior of dynamic systems. Control systems use feedback to adjust the system's output based on the comparison between the desired set point and the actual output. This allows for the correction of errors and disturbances, leading to improved system performance and stability. Examples of feedback mechanisms in engineering include thermostats in HVAC systems, cruise control in automobiles, and automatic voltage regulators in electrical networks [16].

In business and economics, feedback mechanisms are integral to performance management, decision-making, and market dynamics. Feedback from customers, employees, and stakeholders provides valuable information that organizations use to adapt their strategies, products, and operations. In financial markets, feedback mechanisms influence price dynamics, investment decisions, and market efficiency, shaping the behavior of investors and market participants [17].

In social and behavioral sciences, feedback mechanisms are involved in processes such as learning, communication, and social interaction. Feedback plays a crucial role in education, where it is used to provide students with information about their performance, guiding their learning and skill development. In interpersonal relationships and group dynamics, feedback mechanisms influence communication, cooperation, and the evolution of social norms and behaviors [18].

Optimal control is a branch of mathematics that deals with control of a dynamical system with reference to expected output or desired path. This control of a system is done over an interval of time to get the optimal solution with minimum errors [19]. Optimization is applicable in engineering, biology, social and environmental realms, with the use of solar power not exceptional.

Formulation of optimal control function involves the description of a set of state equations which describes the dynamical system (plant), to be controlled. In this study this refers to the control of the drying process of food particles. Because of the physical limitations of the plant, the control variables and the state variables are necessary to describe the constraints of the operation of the plant. The performance index measure which is either time, energy, fuel consumption, or a state regulator problem is modelled in different ways [20].

In this study, the use of feedback mechanism is inevitable, and the necessary controls to obtain desired output is continuous and requires continuous feedback for further adjustment. The problem of solar food dryer involves a state regulation problem, where optimization is done with a concern of making all the drying parameters meet the desired values depending on the nature of food to be dried [21].

3. Dryer Mathematical Model Equations

This section deals with the analysis of heat transfer and mass flow equations necessary in describing the drying process of food particles in the dryer. This is done with a brief description and formulation of a set of equations describing the heat transfer in a heat exchanger, and a set of equations describing the mass flow and energy distribution in the drying chamber. Parameters controlling the efficiency of the heat exchanger and desired drying air characteristics are discussed. The dryer model is subdivided into two sections, namely; the heat exchanger subsystem, and the drying compartment.

A. The Counter flow Heat Exchanger

The heat exchanger is a device used to transfer heat energy from one system of a closed loop of fluid, to another system of fluid at different temperature. In this study, the original source of heat is from a closed loop of hot water, heated from by solar energy, dissipated to the drying air by forced convection using fans fitted to the heat exchanger. The heat exchanger used is a counter flow concentric tube model with copper pipes and steel shell insulated [22], [23].

In this model, the hot fluid $T_{h,in}$ enters at point A while the cold fluid $T_{c,in}$ enters at the opposite end B, and they flow in opposite directions through restricted concentric tubes. This allows for a more uniform temperature difference across the exchanger, resulting in improved heat transfer efficiency. This arrangement is depicted in Figure 1.

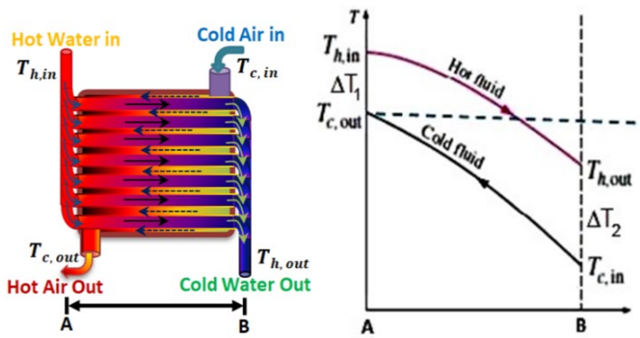

Fig .1. Counter flow heat exchanger configuration, and temperature distribution profile

B. Heat Exchanger Mathematical Model

The mathematical equations of the heat exchanger typically describe the heat transfer and fluid flow characteristics within the exchanger. The calculation of heat transfer across the exchanger is derived from the first law of thermodynamics, which depend on the surface area of the exchanger and the flow rates of the fluid, together with the temperature difference of the two fluids. These equations can include principles of thermodynamics, fluid mechanics, and heat transfer, and are used to analyze and predict the performance of the heat exchanger under different operating conditions. The calculations used in this research assumes that the heat exchanger is of a single-phase type, implying that the entering fluid remains in the same phase as in the outlet.

The energy (E) balance equation is a fundamental principle in thermodynamics that states that the energy entering a system must equal the energy leaving the system, plus any energy accumulated within the system less energy lost in the system. It is expressed mathematically as:

$$E_{In} = E_{Out} + E_{Accumulation} - E_{Lost} \quad (2)$$

This equation is commonly used to analyze and quantify energy transfers in heat exchangers. As the hot fluid transfers some of its energy to the cold fluid, this process is accompanied by respective changes in enthalpy, which can be expressed by the equations for the hot and cold fluids respectively as,

$$\begin{aligned} Q &= -\dot{m}_h c_h dT_h \\ Q &= -\dot{m}_c c_c dT_c \end{aligned} \quad (3)$$

Where $\dot{m} = \rho A \dot{v}$ is the mass flow rate and c the specific heat capacity of the fluid at constant pressure, $A = (width \times length)$ is the surface area of the heat exchanger, and the subscripts h, c denotes the hot and the cold fluid respectively. Using the notation U denoting the heat transfer coefficient, and $\Delta T = T_h - T_c$ for temperature difference, equation (3) for heat transfer can be expressed as,

$$Q = UA\Delta T \quad (4)$$

Where ΔT_m is the mean temperature difference, and U is the heat transfer coefficient given by,

$$U = \frac{1}{\frac{1}{h_o} + \frac{r_o}{K}\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_i}{r_o}\right)\frac{1}{h_i}} \quad (5)$$

Heat transfer for a differential element of length, dx, is obtained by differentiating equation (3) and expressing the temperature difference $d(\Delta T) = dT_h - dT_c$. This yields the relation,

$$\begin{aligned} dQ &= -\dot{m}_h c_h dT_h = -C_h dT_h \\ dQ &= -\dot{m}_c c_c dT_c = -C_c dT_c \end{aligned} \quad (6)$$

where $\dot{m}_h c_h = C_h$ and $\dot{m}_c c_c = C_c$. With little algebra, integration and rearrangements using equation (4), equation (6) can be expressed as,

$$Q = UA\frac{(\Delta T_2 - \Delta T_1)}{\ln(\Delta T_2/\Delta T_1)} \quad (7)$$

Where for counter current, the temperature differences ΔT_1 and ΔT_2 are indicated in Figure 1. and defined as,

$$\Delta T_1 = T_{h,i} - T_{c,o}, \ \Delta T_2 = T_{h,o} - T_{c,i} \quad (8)$$

C. Effectiveness of Heat Exchanger

The effectiveness of a heat exchanger is a measure of how well it is transferring heat between the hot and cold fluids. The general effectiveness of heat exchanger is determined by the ratio of the actual heat transfer to the maximum possible heat transfer, given as,

$$Effectiveness = \frac{Actual\ heat\ Transfer}{Maximum\ possible\ heat\ transfer}$$

In order to express this effectiveness mathematically, let the number of transfer units (NTU) be denoted by N and let $T_{\min}$ and $T_{\max}$ denote the minimum and maximum temperatures respectively. Then the effectiveness of a heat exchanger is a function of heat exchanger surface area A and the heat transfer coefficient U only given by;

$$\epsilon = \frac{Q}{Q_{max}} = \epsilon\left(N, \frac{T_{min}}{T_{max}}\right) \quad (9)$$

where $Q = C_c(T_{c,o} - T_{ci})$ and $Q_{max} = C_c(T_{hi} - T_{ci})$, while $N = \frac{UA}{T_{min}}$.

The rate of heat transfer in a heat exchanger is the amount of heat transferred per unit time between the hot and cold fluids flowing through the exchanger. It is a measure of the heat exchange efficiency and is typically quantified in terms of energy per unit time (e.g., watts or BTU per hour). The rate of heat transfer depends on factors such as the temperature difference between the hot and cold fluids, the surface area available for heat transfer, and the overall heat transfer coefficient of the exchanger. This rate of heat transfer can therefore be described by;

$$Q = \epsilon T_{max} = \epsilon T_{min}\left(T_{h,i} - T_{c,i}\right),$$

where the specific effectiveness of a counter flow heat exchanger is given as in [22] by,

$$\epsilon_{counter} = \frac{1-\exp[-N(1-C)]}{1-C\exp[-N(1-C)]}\,;\; C = \frac{C_{min}}{C_{max}} \quad (10)$$

From the description of the heat exchanger and heat transfer in equations (2) – (9). The output hot air is then channeled through the dehumidifier, (so as to remove any moisture), to the drying chamber, through temperature sensors and flow regulators, so that the desired dry air temperature and flow rate is obtained.

4. Drying Chamber and Water-to-Air Heat Exchanger's Output

The drying chamber is the final and major target of the entire food drying model. The other subsystems are adjusted to suit the requirements of the dryer. It is in the drying chamber that food products are placed, and exposed to hot drying air for dehydration depending on their nature.

The drying chamber are equipped with heat exchangers placed on the walls of the drying chamber, each fitted with controlled fans, to regulate the air mass flow rate.

Depending on the type of food, amount of food to be dried, their initial moisture content, the required final moisture content, and the expected urgency of the drying process (or residence time), adjustment is made on the hot air mass flow rate, thermal liquid mass flow rate. These parameters are continuously controlled using a feedback mechanism for the entire time that food products are undergoing dehydration process.

Various food products have their upper limit of maximum temperature that can be exposed to dehydrate, of which, beyond this limit, the food products will start getting cooked or roasted, thus spoiling their quality. The food particles to be dried are placed in trays in the drying chamber, and hot dry air is channeled from the heat exchangers through the trays to eliminate moist air in the dryer. Measuring devices are placed to monitor the temperature, time, mass flow rate, number of radiators switched on, among other variables.

For a water to air heat exchanger, the temperature of the extracted hot air from the heat exchanger depends on the variables which include; input temperature of thermal liquid $T_{h,i}$, air mass velocity through the fan v_a, sweep area of the fan A_f, water volumetric flow rate $\dot{V}_w$, heat transfer area of the exchanger A_e and heat exchangers' coefficient of heat transfer U. For a fixed area of heat exchanger, with fixed input thermal fluid temperature, and fixed fan area, the only variable used to control the output temperature, is the air mass flow rate through the fan v_a. Incorporating this into equation (8) and (9) yields,

$$N = \frac{UA}{\rho_a A_f c_a v} := \frac{K}{v} \text{ where } K = \frac{UA}{\rho_a A_f c_a} \quad (11)$$

And thus, the effectiveness as a function of air velocity is defines as,

$$\epsilon(v)_{counter} = \frac{1-e^{-\frac{K}{v}(1-C(v))}}{1-C(v)\,e^{-\frac{K}{v}(1-C(v))}} \quad (12)$$

Where $C(v) = \frac{C_{min}(v)}{C_{max}(v)}$, and if C_{min} is the air side, then $C(v) = \frac{\rho_a A_f v c_a}{\rho_h \dot{V}_h c_h}$ and the output temperature of air is given by,

$$T_{a,o} = T_{a,i} + \epsilon(v)\left(T_{h,i} - T_{a,i}\right) \quad (13)$$

Where the subscript a- air, i – in, o- out and h for hot water. Using the food drier model data as presented in Table 1, the output temperature of hot air can be illustrated as shown in Figure 2.

Table 1
Concurrent heat exchanger simulation variables

Variable	Symbol	Value
Hot water inlet	$T_{h,i}$	100 ^{0}C
Hot water flow rate	$\dot{V}_h$	1.128 lit/s
Density of water	ρ_h	997 Kg/m^3
Specific heat capacity of water	c_h	4186 J/Kg-K
Air in temperature	$T_{a,i}$	25 ^{0}C
Fan area	A_f	0.05m^2
Air velocity	v_a	4.0m/s
Density of air	ρ_a	1005J/Kg-K
Heat transfer coefficient	U	100W/m^2-K
Exchanger heat transfer Surface area	A	2.827m^2
Cold air inlet temp	$T_{a,i}$	25^{0}C

From Figure 2, the temperature of the thermal fluid drops slightly, and the output air temperature is seen to increase from the initial input temperature of 25^{0}C to 75.6^{0}C as illustrated. This temperature however is obtained at constant air flow rate and constant fan revolution per minute. Any changes to these variables affect the output temperature of the drying air.

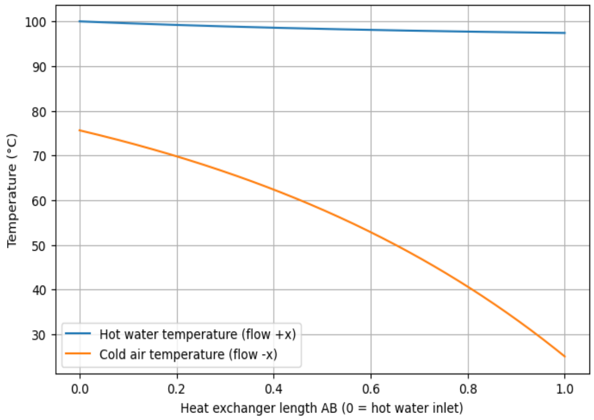

Fig. 2. Concurrent heat exchanger hot air output temperature profile

The parameters related to the output in Figure 2 are presented in Table 2. These include; NTU of $N = 1.172$, capacity ratio $C = 0.051$, heat exchanger's effectiveness $\epsilon(v) = 0.6826$ and the maximum air temperature of $T_{a,o} = 75.6^0C$. These values are achieved using the input values presented in Table 1.

Table 2
Heat exchanger simulation output parameters

Variable	Symbol	Value
NTU	N	1.172
Capacity ratio	C	0.051
Effectiveness	$\epsilon(v)$	0.6826
Air outlet temperature	$T_{a,o}$	75.6 ^{0}C

A. Air Mass flow rate and Output Temperature

The optimal output temperature of the drying air is obtained by tunning the variables of the fan area, fan speed and air flow velocity. It is noted that these tree variables have inverse relationship with the output temperature as shown in Figure 3.

Simulation of the effect of air velocity, fan area and fan rotation speed are presented in Figure 3.

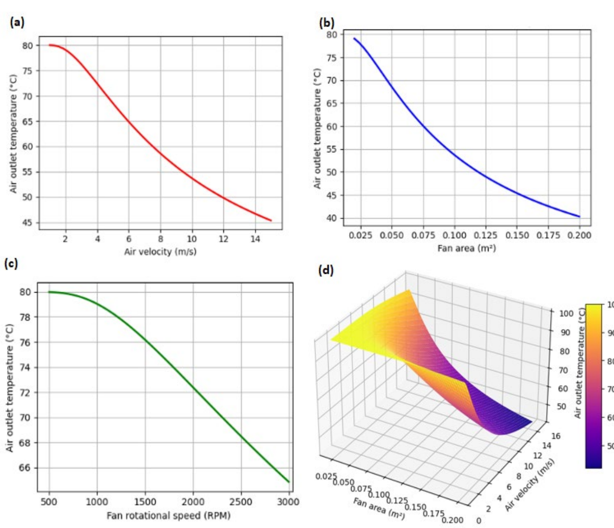

Fig. 3. Effect of air velocity, fan sweep area and fan speed on output air temperature

It is noted that all the three variables are inversely proportional to the output temperature. At low air velocity, flowing air through the heat exchanger will have adequate time to pick heat from the hot pipes (heated by hot water) and therefore the lower the velocity, the higher the output temperature of the drying air as indicated in part (a) of Figure 3. Similarly, bigger diameter of a fan sweeps a larger area forcing more air through the heat exchanger, thus end up cooling the heat expected to be output. The smaller the amount of air flowing through the heat exchanger increases the enthalpy and thus brings out more heat. This is illustrated in part (b) of Figure 3. The third variable on fan rotation speed becomes obvious from the linear relation of fan speed and air velocity $v = k_n N$, where v denotes air velocity, N fan rotation speed and k_n variation constant, that increase in fan speed lowers the output air temperature as depicted in Figure 3(c). Putting all of them together, a surface plot representing the relation of all the three variables is presented in Figure 3(d).

B. Optimization of Drying Air Temperature

An optimization problem typically involves maximizing or minimizing a certain objective function while satisfying a set of constraints. Such a problem can be solved using mathematical techniques such as linear programming, nonlinear programming, and evolutionary algorithms, multi-objective particle swarm optimization, non-dominated sorting genetic algorithms, pareto-front based simulated annealing, Pontryagins' methods, among others. The goal of solving an optimization problem is to identify the most efficient or effective solution given a specific set of constraints (conditions are essential for defining the boundaries or limitations within which the optimization problem must be solved) and maximization or minimization of desired objective(s) of interest restricted by constraints with attainable values.

The Pontryagin's Minimum Principle (PMP) is a key concept in the field of optimal control theory, particularly in the context of continuous-time systems. It provides necessary conditions for optimality in problems involving the minimization of a cost function, subject to differential equations that describe the dynamics of the system. The associated optimization algorithm, known as the Pontryagin's Minimum Principle, is used to solve such optimal control problems by determining the control inputs that minimize the specified cost function while satisfying the system dynamics [24].

The study proposes to use the Pontryagin's maximum principle to solve the optimal control problem involving the decision variables $x(t)$ and the objective variable $u(t)$. This involves the process of determining control and state trajectories for a dynamic system over a period of time in order to minimize a performance index.

From equation (3), we have heat energy equal to the local heat flux through the wall, and thus in an infinitesimal length element dx, we have;

$$\delta \dot{Q} = UA(T_h - T_c)dx \tag{14}$$

Expressing equation (14) in terms of equation (3) yields;

$$\frac{dT_h}{dx} = -K_h(T_h - T_c)$$
$$\frac{dT_c}{dx} = -K_c(T_h - T_c) \quad (15)$$

With the negative sign denoting that the heat is increasing in the opposite direction of the x – axis in a countercurrent heat exchanger arrangement, where $K_c = \frac{UA}{\dot{m}_c c_c}$ and $K_h = \frac{UA}{\dot{m}_h c_h}$, and other parameters carry the same meaning.

To maximize the outlet air temperature $T_{c,o}$, given inlet hot water temperature $T_{h,i}$, this forms an optimal control problem with the controls $u_1(t)$ denoting air velocity which affects the air mass flow rate $\dot{m}_a(u_1)$ and $u_2(t)$ denoting the fan speed which affects both air velocity and convective heat transfer coefficient $h_a(u_1, u_2)$.

Define the variables $x = (x_1, x_2)$ as,

$$x_1(t) = T_h(t),\ x_2(t) = T_c(t) \quad (16)$$

Representing the water temperature and air temperature respectively. The state equation is therefore presented as;

$$\dot{x_1}(t) = -K_h(u_2)(x_1 - x_2)$$
$$\dot{x_2}(t) = -K_c(u_2)(x_1 - x_2) \quad (17)$$

The objective function of this optimal control problem is to maximize the output air temperature $T_{c,o}$ while penalizing the power used by the fan in terms of fan power u_2^2 and fan resistance u_1^2. This yields the objective function,

$$J(x, u) = x_2(t_f) - \int_o^{t_f}(c_1u_1^2 + c_2u_2^2)dt \quad (18)$$

Where t_f is the optimality feasible time limit, c_1, c_2 are arbitrary weighting constants representing energy cost coefficients. $J(x, u)$ describes the performance index, which in this case describes how best the controls can be manipulated to give the maximum temperature. The main objective of this Optimal Control problem is finding the piecewise continuous control $u(t) = (u_1, u_2)$ and the associated state variable $x(t) = (x_1, x_2)$ to maximize the given objective functional $J(x, u)$.

In order to solve this optimal control problem in equation (17 – 18), we will apply Pontryagin's Maximum Principle.

Pontryagin's Maximum Principle is a mathematical theory used to solve optimization problems for dynamic systems, particularly in control theory and optimal control. In this problem, we define the adjoint functional H here referred to as Hamiltonian as;

The Hamiltonian, named after the Irish mathematician William Rowan Hamilton, is a fundamental concept in classical mechanics and mathematical physics [25]. It is a scalar function that summarizes the dynamics of a physical system, particularly in the context of conservative systems.

The Hamiltonian is hereby defined by;

$$H(t, x(t), u(t), \lambda(t)) = \lambda_1\dot{x_1} + \lambda_2\dot{x_2} - (c_1u_1^2 + c_2u_2^2) \quad (19)$$

Here, the parameter $\lambda(t)$ denotes the multiplier of the state variable. Substituting equation (17) into equation (19) yields,

$$H(t, x(t), u(t), \lambda(t)) = -\lambda_1K_h(u_2)(x_1 - x_2) - \lambda_2K_c(u_2)(x_1 - x_2) - (c_1u_1^2 + c_2u_2^2) \quad (20)$$

Which can be simplified to;

$$H = U(u_2)(x_1 - x_2)[-\lambda_1A_1 + \lambda_2A_2] - (c_1u_1^2 + c_2u_2^2) \quad (21)$$

Where $A_1 = \frac{A}{\dot{m}_h c_h}$, $A_2 = \frac{A}{\dot{m}_c(u_1)c_c}$ are redefined to emphasize the function of U and the effect of u_1 on $\dot{m}_c$.

Solving to minimize the Hamiltonian in equation (21) is done by evaluating the condition which satisfies the equations,

$$\frac{\partial H}{\partial u} = 0 \quad (22)$$

That is,

$$\frac{\partial H}{\partial u_1} = U(u_2)(x_1 - x_2)\lambda_2\frac{\partial A_w}{\partial u_1} - 2c_1u_1$$
$$\frac{\partial H}{\partial u_2} = (x_1 - x_2)[-\lambda_1A_1 + \lambda_2A_2]\frac{dU}{du_2} - 2c_2u_2 \quad (23)$$

Solving equation (22) in terms of equation (23) yields the optimal fan speed u_1^* and optimal air velocity u_2^* as,

$$u_1^* = \frac{U(u_2)(x_1-x_2)\lambda_2}{2c_1}\frac{\partial A_2}{\partial u_1}$$
$$u_2^* = \frac{(x_1-x_2)[\lambda_1A_1+\lambda_2A_2(u_i)]}{2c_2}\frac{dU}{du_2} \quad (24)$$

Where $U(u_2)$ is the relation between overall conductance and fan speed, which could be linear with k_u as the sensitivity of the slope.

Using the data in Table 1. The simulation of optimal air velocity and optimal fan speed is illustrated in Figure 4.

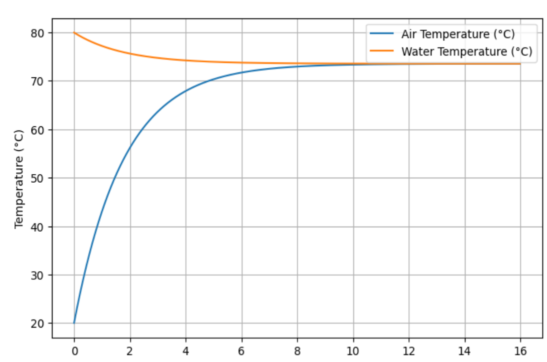

Fig. 4. Optimal control of heat exchanger (Pontryagin)

The optimal values are $v_{min} = 0.2m/s$ air velocity and using $v = k_nN$, with $k_n = 0.01$, the fan speed will be $N = 20\ RPS$. It in noted from Figure 5 that the optimal output temperature of hot drying air is 75.6^0C, achieved after 10 seconds of simulation. This is based on the condition that the input water temperature into the heat exchanger is maintained at 80^0C throughout the simulation process.

5. Conclusion and Recommendations

The use of countercurrent water to air heat exchanger to produce hot air, can be used to generate hot air sufficient to dry food products to desired moisture content.

In this research study, focus was on the dryer, parameter range and optimization of alternative source of energy. The regulation of outlet air temperature in water–air heat exchangers play a pivotal role in enhancing the thermal performance and operational efficiency of drying systems. This work has formulated the temperature control task as an optimal control problem, employing Pontryagin's Maximum Principle to derive a principled strategy for adjusting air velocity and fan speed. The resulting framework enables maximization of heat transfer while minimizing energy expenditure, offering a dynamic alternative to static control schemes.

Simulation results indicate that adaptive control trajectories yield superior thermal stability and responsiveness under varying process conditions. The counterflow configuration of the heat exchanger, in conjunction with the Hamiltonian-based optimization, underscores the sensitivity of outlet air temperature to both hydraulic and aerodynamic parameters. These findings reinforce the necessity of real-time control in achieving consistent drying performance.

The methodology presented herein provides a foundation for the development of intelligent drying systems that balance energy efficiency, product quality, and cost-effectiveness. Future investigations may extend this framework to incorporate moisture-dependent transport phenomena, nonlinear coupling between heat and mass transfer, and model-predictive control architectures. Such enhancements are expected to further improve the fidelity and robustness of temperature regulation in complex drying environments.

References

[1] L.-S. Wang, *A Treatise of Heat and Energy*. Cham, Switzerland: Springer, 2020.

[2] M. H. Bollen and F. Hassan, *Integration of Distributed Generation in the Power System*. Hoboken, NJ, USA: John Wiley & Sons, 2011.

[3] M. Asplund, N. Grevesse, A. J. Sauval, and P. Scott, "The chemical composition of the Sun," *arXiv preprint* arXiv:0909.0948, 2009.

[4] R. Sims, P. E. Mercado, and W. Krewitt, "Integration of renewable energy into present and future energy systems," 2011.

[5] J. V. Fenhann, P. Sommer-Larsen, and S. Furbo, "Energy technologies for smart cities: Solar energy," in *Risø Energy Report 10: Energy for Smart Cities in an Urbanised World*. Roskilde, Denmark: Technical University of Denmark, Risø National Laboratory for Sustainable Energy, 2011, pp. 41–45.

[6] S. D. Tung, "Heat pumps for sustainable heating and cooling," 2020.

[7] S. van Herwaarden, "Calorimetry measurement," in *Mechanical Variables Measurement—Solid, Fluid, and Thermal*. Boca Raton, FL, USA: CRC Press, 2023, pp. 17-1–17-16.

[8] J. Buzás and I. Farkas, "Solar domestic hot water system simulation using block-oriented software," in *Proc. 3rd ISES-Europe Solar Congress (EuroSun 2000)*, Copenhagen, Denmark, 2000, p. 9.

[9] A. N. Neto, "Dryer modeling and optimization," Ph.D. dissertation, Texas Tech Univ., Lubbock, TX, USA, 1997.

[10] X. D. Chen and A. S. Mujumdar, *Drying Technologies in Food Processing*. Hoboken, NJ, USA: John Wiley & Sons, 2009.

[11] D. Xiao, C. and S. Arun, M., "Drying technology and food processing," *Int. J. Food Nutr. Safety*, 2008.

[12] N. Ahmed, J. Singh, H. Chauhan, P. G. A. Anjum, and H. Kour, "Different drying methods: their applications and recent advances," *Int. J. Food Nutr. Safety*, vol. 4, no. 1, pp. 34–42, 2013.

[13] A. Haghi, "A mathematical model of the drying process," *Acta Polytechnica*, vol. 41, no. 3, 2001, doi: 10.14311/226.

[14] A. Iguaz, A. Esnoz, G. Martínez, A. López, and P. Virseda, "Mathematical modelling and simulation for the drying process of vegetable wholesale by-products in a rotary dryer," *J. Food Eng.*, vol. 59, no. 2–3, pp. 151–160, 2003.

[15] C. Cosentino and D. Bates, *Feedback Control in Systems Biology*. Boca Raton, FL, USA: CRC Press, 2011.

[16] J. M. T. Thompson and H. B. Stewart, *Nonlinear Dynamics and Chaos*. Chichester, UK: John Wiley & Sons, 2002.

[17] L. Goncharenko, E. Sharko, S. Sybachin, M. Khachaturyan, and Z. Prokopenko, "Feedback mechanism of digital technology using strategies of enterprises," in *The Social Sciences Empowered*. Boca Raton, FL, USA: CRC Press, 2020, pp. 65–70.

[18] G. W. McLeod and R. J. Mortimer, "Evaluating feedback mechanisms in the School of Earth and Environment," *Planet*, vol. 26, no. 1, pp. 36–41, 2012.

[19] S. Lenhart and J. T. Workman, *Optimal Control Applied to Biological Models*. Boca Raton, FL, USA: Chapman and Hall/CRC, 2007.

[20] F. Lin, *Robust Control Design: An Optimal Control Approach*. Chichester, UK: John Wiley & Sons, 2007.

[21] F. L. Lewis, D. Vrabie, and V. L. Syrmos, *Optimal Control*. Hoboken, NJ, USA: John Wiley & Sons, 2012.

[22] M. Boda, S. Deshetti, and M. Gavade, "Design and development of parallel-counter flow heat exchanger," *Int. J. Innov. Res. Adv. Eng.*, vol. 4, no. 2, pp. 29–35, 2017.

[23] W. Kim, Y.-J. Baik, S. Jeon, D. Jeon, and C. Byon, "A mathematical correlation for predicting the thermal performance of cross, parallel, and counterflow PCHEs," *Int. J. Heat Mass Transfer*, vol. 106, pp. 1294–1302, 2017.

[24] L. S. Pontryagin, *Mathematical Theory of Optimal Processes*. Boca Raton, FL, USA: CRC Press, 1987.

[25] Z.-C. Yang, A. Rahmani, A. Shabani, H. Neven, and C. Chamon, "Optimizing variational quantum algorithms using Pontryagin's minimum principle," *Phys. Rev. X*, vol. 7, no. 2, p. 021027, 2017.