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Measuring the Technical Efficiency of Educational Units – An Interesting Find

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***Abstract*: This paper empirically investigates the existence of differences in the measurements in Technical Efficiency for upper secondary public schools in Greece during the years 2020, 2021 and 2022 with implementation of Data Envelopment Analysis and Stochastic Frontier Analysis. The findings show that the results are close. The mean value of the differences (d) is 0.00022, and the standard deviation is 0.083. The application of SFA is performed with the Cobb-Douglas model, Half-Normal $N^{+}(0, \sigma_u^2)$, which was chosen among four alternatives, since it presents the best adaptation to the empirical data. Therefore, in the area of applied policy there is no need to apply both methods.**

***Keywords*: Upper Secondary Schools, Technical Efficiency, Data Envelopment Analysis, Stochastic Frontier Analysis, Environmental Variables.**

1. Introduction

According to economics the educational units, in order, to carry out their mission and task, consume resources from the limited and competitively claimed and provide a variety of outputs, outcomes, benefits, positive externalities and spillovers to the pupils/students, economy and society. Optimal allocation and use of resources by educational units to achieve their goals are critical issues for education and the economy [Tsamadias (2020)].

Evaluating the technical efficiency and consequent improvement interventions of the educational units can lead to a reduction in the waste of the used resources and consequently to an increase in the attainments.

The international theoretical and empirical literature on the above topics has been extensive in recent decades.

The methods-techniques commonly used to measure the efficiency of educational units are Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA). The DEA method estimate the relative efficiency among homogeneous DMUs (here USSs) that use similar resources (inputs), have the same technology to pursue similar objectives (outputs). DEA is a mathematical programming approach. Has its origins in the work of Charnes et al. (1978) who reformulated Farrell's (1957) seminal work. SFA is a stochastic technique which contains a random error term. It has its roots in his work Aigner and Chu (1968), Aigner et al. (1977), and Meeusen and van den Broeck (1977) [Coelli et al. (2005)].

Greece is a country in Southeastern Europe (comprises a total area of 131,957 km^2 and has a population of almost 11 million inhabitants), member of European Union and Eurozone. The region of Central Greece (area of 15,549 km^2(11.8%) and a population of 547,390 (5.07%) inhabitants), is a representative of the 13 country's regions since the main economic, social and educational characteristics that most of them have, are about on average the same [Karatheodoros et al. (2016)].

Secondary education is divided into two levels, the lower compulsory that is provided in the Lower secondary schools, day and evening and is of three-year duration, and the higher non-compulsory provided in the Upper secondary schools and is distinguished: a. In general, provided in General High schools, b. In the vocational, provided in the Vocational High Schools. Admission to universities is done by nationwide examinations of candidates. In region of Central Greece there are 64 public homogeneous USSs[1] that operate during the period under review.

The purpose of this paper is to empirically investigate if the two methods, DEA and SFA, give the similar results of Technical Efficiency (TE), of the examined schools. Additionally, to investigate the effect of environmental variables on possible differences between results of two methods. To the best of our knowledge this is the first study that examines a statistical process by which the most appropriate model used with the SFA method is selected.

The rest of the study is organized as follows: Section 2, provides a brief review of theoretical and empirical literature. Section 3, presents the empirical analysis (variables, sampling, sources, data and descriptive statistics, measuring efficiency–results, the effect of environmental variables on different average, a short discussion). Finally, Section 4 presents the final conclusions and policy proposals.

[1] USSs: Upper Secondary Schools

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2. Review of Literature

In the international theoretical and empirical literature, for evaluating technical efficiency, there are two prevailing methodologies: DEA and SFA. The review of empirical studies shows that 86.66 % of studies use DEA and 11.66% use SFA [Margaritis et al. (2020)].

A. Data Envelopment Analysis

DEA is a non-parametric method. The current paper uses DEA with Orientation to Outputs (Output Oriented / OO) (in the OO model, inputs are fixed and units maximize the level of outputs (Agasisti and Pérez-Esparrells, 2010) under the hypothesis variable returns to scale which helps to estimate efficiently whether an increase or decrease in inputs or outputs does not result in a proportional change in the outputs or inputs respectively (Cooper et al, 2011), so that the results are comparable to SFA.

The DEA-TE-OO-with VRS assumption is the following linear programming problem which is for each unit is solved (Banker et al, 1984; Johnes, 2006):

Maximize: θ

Subject to

$$x_{ij0} - \sum_{j=1}^{t} \lambda_j x_{ij} \geq 0; i = 1,2,...,m; j = 1,2,...,t\,,$$

$$\theta x_{ij0} - \sum_{j=1}^{t} \lambda_j y_{rj} \geq 0; r = 1,2,...,s; j = 1,2,...,t\,,$$

$$\sum_{j=1}^{n} \lambda_j = 1\,; j = 1,2,...,t\,,\ \lambda_j \geq 0; j = 1,2,...,n.$$

where θ:degree of TE, m: numbers of outputs and n: number of inputs, t: number of DMUs to be measured, $x_{ij0:}$ amount of input i for the j_0 required to measure its TE, y_{rj}: amount of output r for the DMU required to measure its TE, λ_j: weight of input and output of the DMU j.

B. Stochastic Frontier Analysis

SFA is a parametric method, proposed by Aigner et al. (1977) and by Meeusen et al. (1977) and extended with the introduction of other models by Battese and Coelli (1995). The inclusion of random errors in the equation of the unknown production limit in combination with appropriate assumptions for the distribution of random variables expressing the errors and the form of the production function are the main features of the SFA method. The general model of the contemplative frontier method is of the form $\ln Y_i = f(b, X_i) + V_i - U_i$, i = 1,2, .., N, where $\ln Y_i$ is the natural logarithm of the output Y_i of the unit i, X_i is the vector input of the unit i, b is the vector of the unknown factors, V_i is the symmetrical component of the composite error $\mathcal{E}_i = V_i - U_i$, U_i is the component that expresses the inefficiency and $f(b, X_i)$ a function of b and X_i. For the variable V_i we accept that it follows the normal distribution $N(0, \sigma_v^2)$ with mean value zero and variance σ_v^2. For Ui we accept that it is independent of V_i, non-negative and follows either the semi-normal distribution $N^+(0, \sigma_u^2)$ or the zero-divided normal distribution $N^+(\mu, \sigma_u^2)$ or the exponential with parameter $1 / \sigma_u$ or even gamma G (λ, m) with mean λ and m degrees of freedom. Also, the two variables V_i and U_i are considered independent of the regressors X_i. The output function $f(b, X_i)$ in SFA models is usually Cobb-Douglas(C-D) or Translog (Tr) (Transcendental Logarithmic), with equations (2) and (3) respectively:

$$\ln Y_i = b_0 + b_1 \ln X_{1i} + b_2 \ln X_{2i} + \ldots + b_k \ln X_{ki} + V_i - U_i \qquad (2)$$

$$\ln Y_i = b_0 + b_1 \ln X_{1i} + b_2 \ln X_{2i} + \ldots + b_k \ln X_{ki} + \left(\frac{1}{2}\right) \sum\nolimits_{n=1}^{k} \sum\nolimits_{m=1}^{k} \mathbf{b_{nm}} \ln X_{ni} \ln X_{mi} + V_i - U_i^2 \qquad (3)$$

for i = 1,2,…,N. For the β_{nm} coefficients of the interactive terms $\ln X_{ni}$, $\ln X_{mi}$ we assume that $b_{nm} = b_{mn}$ applies.

Combining each of the above equations with the two commonly used distributions, semi-normal and reduced to zero, the normal distribution for the variable Ui variable results in four SFA models for measuring efficiency. In all these models the technical efficiency TE_i of a unit i is estimated by $TE_i = \exp(-U_i)$, more precisely than their average value that is $TE_i = E[\exp(-U_i)]$. TE_i's estimate therefore requires estimating the coefficients $b_0, b_1, \ldots, b_k$, k + 1 for Cobb-Douglas models and $b_0, b_1, \ldots, b_k, b_{12}, \ldots, b_{kk}$, multitude (1 / 2) (k + 1) (k + 2) for the Translog models and the scatter parameters σ_v^2 and σ_u^2. This is achieved with the help of the logarithm-probability function, which includes the above coefficients and the parameters. Instead of this parameterization, Battese and Corra (1977) suggested replacing σ_v^2 and σ_u^2 with $\sigma^2 = \sigma_v^2 + \sigma_u^2$ and $\gamma = \sigma_u^2 / (\sigma_v^2 + \sigma_u^2)$. The parameter γ takes values from zero(0) to one(1) and expresses the percentage of dispersion of the variable U inefficiency in relation to the total dispersion of U_i and V_i. The adaptation of the above two models, C-D and Tr to the respective data, is generally different. Appropriate statistics check on the one hand which of the C-D or Tr models shows the best fit and which of the above Ui distributions will be used [Coelli et al., 2005; Ferdous F.K. et al., 2011; Scippacercola S., D'Ambra L., 2014]. Finally, parameter control γ, statistically confirms the degree of efficiency problems between production units. Translog function is very commonly used- it is a generalization of the Cobb-Douglas function.

C. Empirical Studies

Published works include empirical studies which assess the TE of several real-world homogeneous production systems using both DEA και SFA.

Dairy industry (Reinhard et al.(2000)); *Healthcare system* (Mortimer, D.(2002); Jacobs(2001); Giokas, D.(2001); Assaf, A and Matawie, K.(2008); Lee, et al.(2009); Kontodimopoulos

[2] V_i: noise error term-symmetric (eg. normal distribution) U_i: inefficiency error term-non-negative (eg.half-normal distribution)

Table 1
Empirical studies (field of Education) which measure efficiency with the methods DEA & SFA

Authors	Country	Field	Sectors	Methodology(means) DEA	SFA
Ruggiero, J. and Vitaliano, F. D. (1999)	USA	Education	Public elementary and secondary districts	0.86	0.86
Chakraborty,K., Biswas, B. and Lewis, C. W. (2001)	USA	Education	Public secondary districts	0.861	0.885
Mizala, A., et al. (2002)	Chile	Education	Public and private (fee-paying & subsidized) schools	0.93	0.86
G. Thomas Sav (2012)	USA	Education	Colleges	0.56	0.45
Scippacercola, S. and D'Ambra, L. (2014)	Italy	Education	Public secondary schools	0.85	0.84
Rzadzinski, L. and Sworowska, A. (2016)	Poland	Education	Higher Vocational Schools	0.96/0.82/0.94	0.99/0.54/0.70

Table 2
Descriptive statistics of inputs and output variables by year

Variables/Statistics	Inputs X_1	X_2	X_3	X_4	Output Y_1
		2020			
AVG	165	15.74	420,566	21.63	39.77
S.D.	101	7.46	218,251	9.31	28.2
Max	442	36	932,105	44	117
Min	34	5	110,629	5	5
		2021			
AVG	165.3	15.78	408,349	21.63	40.23
S.D.	103.3	7.62	197,665	9.31	29.11
Max	436	37	882,873	44	136
Min	35	5	117,223	5	5
		2022			
AVG	160.4	15.50	417,327	21.63	44.13
S.D.	100.5	7.22	198,832	9.31	32.59
Max	426	37	914,644	44	129
Min	34	4	113,336	5	7

Source: Author's calculation
Notes: 1. AVG: Average, 2. S.D.: Standard Deviation

et al.(2010); Varabyova, Y. and SchreyOgg, J.(2013); Katharakis, G. et al.(2014); Novignon, J. and Lawanson, A.(2017)); *Bank system* (Fiorentino, E. et al.(2006); Kuchler, A.(2013); Silva, C., T. et al.(2017); Lai-Wang Wang et al.(2019); Nguyen, Ph. and Pham, D.(2020)); *Primary care trusts (PCTs)* (Martin, S., Smith, P.(2010)); *Agricultural sector-farms* (Theodoridis, A. and Mazgarul, A.(2011); Madau, Fabio A.(2012); Zamanian, Gh., et al.(2012); Umar, H., S. et al.(2018); Jun-Yen Lee(2005)); *Public Transportation* (Margari et al.(2007); Marcus Vinicius Pereira de Souza et al.(2009); Kuosmanen, T., Saastamoinen, A. and Sipiilainen, T.(2013); Scippacercola, S. and Sepe, E.(2014)); *Largest Syndicates* (Milton N. et al.(2015)); *Machinery Industry*(Yan Xiong, Zhidong Li & Xi Fang (2017)); *Container ports*(Hlali, A.(2018)); *Water Utilities* (Murwirapachena, G., et al. (2019); Parman, B., and Featherstone, A.(2019); Menzies, N., et al. (2020)); *Sugar industries*(Robabeh Ghayeghran Sarab, Alireza Amirteimoori, Alaeddin Malek & Sohrab Kordrostami (2021)). None of the above selects the best fit of the empirical data. Furthermore, a number of studies have been conducted in the education field, which are shown in Table 1.

And in these papers concerning the field of education, the SFA is not applied to any of them by choosing the model that presents the best adaptation to the empirical data.

It is also worth noting that in none of the aforementioned publications SFA with best fit to empirical data is used. Further references are:

1. (Sotiriadis et al. (2015)) which examines the TE of the 92 USSs in the Region of Central Macedonia for the years 2007-08 and 2010-11. This study estimates by the application of DEA IO-VRS the average TE at the 0.814 και 0.835 level respectively.
2. (Margaritis et al. (2020)) which examines the TE of the 64 USSs of the region of Central Greece during the period 2015–2018. This study estimates by the application of DEA IO-VRS the average TE at the 0.936 level.

3. Empirical Analysis

In this study, the technical efficiency of the examined 64 public school units of the upper secondary education is measured using: i. The non-parametric method DEA VRS (OO) and ii. The SFA parametric method.

A. Variables, Sampling, Sources, Data and Descriptive Statistics

This paper uses 4 inputs for each school unit: X_1: Number of students, X_2: Number of teachers, X_3: Public expenditure (€), X_4: Number of computers, and an output Y_1: Number of students who graduated (passed national exams for the first time) and entered the country's Universities. The data for the inputs and outputs for the years 2020, 2021, 2022 come from competent services of the country and the Region.

The Table 2 below presents the descriptive statistics of the four inputs and one output for the years 2020, 2021 and 2022 of the examined school units.

The average number of students per USS is approximately 165. The average number of students per teacher is approximately 16. The average total expenditures per USS is approximately 413,600€. The average number of students per computer is approximately 8.

Selection of the appropriate model for the implementation of the SFA.

To select the appropriate model among the 4 alternatives [C-D, Half-Normal, $N^+(0, \sigma_u^2)$]; [C-D, Truncated Normal, $N^+(\mu, \sigma_u^2)$]; [Translog, Half-Normal, $N^+(0, \sigma_u^2)$]; [Translog, Truncated Normal, $N^+(\mu, \sigma_u^2)$], we perform the following tests/steps.

Step 1: *We test hypotheses*: H_0: $\mu = 0$ vs H_1: $\mu \neq 0$. C-D, Half-Normal, $N^+(0, \sigma_u^2)$]

The test will give us the statistical information on which of the above two distributions U_i follows. This test will be performed with the help of the statistical function $Z = \hat{\mu}/se(\hat{\mu})$ that follows (large size 64> 30) the standard normal distribution N(0,1). From table A.4, we find that the value of the criterion $Z = \hat{\mu}/se(\hat{\mu})$ is Z =-0.50901434 whose absolute value do not exceeds the critical value $Z_{0.975}$ = 1.96 so *we do not reject* the null hypothesis that the half-normal model is adequate (at the 5% level of significance) [Coelli et al., 2005]. Alternatively, from the maximised log-likelihood values reported in Table A.4, we find that the value of the statistical function LR = -2 (LnL_R - LnL_U) =0.5663618. The LR statistic follows approximately, the χ^2 distribution (more precisely a mixed χ^2) with a degree of freedom. As a critical value for the control at the level of significance α = 5% we take $\chi^2_{0.95}(1)$ = 3.84 which isn't less than the value of LR. Thus, the LR test leads us to *do not reject* the null hypothesis that the half-normal model isn't adequate (at the 5% level of significance). *Therefore, in the model, the null hypothesis is true.*

[C-D, Truncated Normal, $N^+(\mu, \sigma_u^2)$]

From the maximised log-likelihood values reported in Table A.5, we find that the value of the statistical function LR =-2(LnL_R - LnL_U) =0.5663618. The LR statistic follows approximately, the χ^2 distribution (more precisely a mixed χ^2) with a degree of freedom. As a critical value for the control at the level of significance α = 5% we take $\chi^2_{0.95}(1)$ = 3.84 which isn't less than the value of LR [Coelli et al., 2005]. Thus, the LR test leads us to *do not reject* the null hypothesis that the half-normal model isn't adequate (at the 5% level of significance). *Therefore, in the model, the null hypothesis is true.*

[Translog, Half-Normal, $N^+(0, \sigma_u^2)$]

This test will be performed with the help of the statistical function $Z = \hat{\mu}/se(\hat{\mu})$ that follows (large size 64> 30) the standard normal distribution N(0, 1). From table A.6, we find that the value of the criterion $Z = \hat{\mu}/se(\hat{\mu})$ is Z = -1.20090146 whose absolute value do not exceeds the critical value $Z_{0.975}$ = 1.96 so *we do not reject* the null hypothesis that the half-normal model isn't adequate (at the 5% level of significance) [Coelli et al., 2005]. Alternatively, from the maximised log-likelihood values reported in Table A.6, we find that the value of the statistical function LR =-2(LnL_R-LnL_U) =0.1668956. The LR statistic follows approximately, the χ^2 distribution (more precisely a mixed χ^2) with a degree of freedom. As a critical value for the control at the level of significance α = 5% we take $\chi^2_{0.95}(1)$ = 3.84 which isn't less than the value of LR. Thus, the LR test leads us to *do not reject* the null hypothesis that the half-normal model isn't adequate (at the 5% level of significance). *Therefore, in the model, the null hypothesis is true.*

[Translog, Truncated Normal, $N^+(\mu, \sigma_u^2)$]

From the maximised log-likelihood values, we find that the value of the statistical function LR = -2 (LnL_R - LnL_U) =0.1668956. The LR statistic follows approximately, the χ^2 distribution (more precisely a mixed χ^2) with a degree of freedom. As a critical value for the control at the level of significance α = 5% we take $\chi^2_{0.95}(1)$ = 3.84 which isn't less than the value of LR. Thus, the LR test leads us to *do not reject* the null hypothesis that the half-normal model isn't adequate (at the 5% level of significance).

Therefore in the model, the null hypothesis is true.

Furthermore, in all four models, the null hypothesis is not rejected, which means that the statistically proposed model is one of the models:[C-D, Half-Normal, $N^+(0, \sigma_u^2)$] or [Translog, Half-Normal, $N^+(0, \sigma_u^2)$] *(at the 5% level of significance).*

Step 2: *We test hypotheses*: H_0: $b_{11}=b_{12}=b_{13}=\ldots=b_{34}=0$ vs H_1: H_0 is not valid.

In order to statistically determine which of these two models is preferable, from the specific data. From the maximised log-likelihood values we find that the value of the statistical function LR = -2 (LnL_R - LnL_U) =-2(-3.9800427-2.5274026)=13.0148906. The LR statistic follows approximately, the χ^2 distribution (more precisely a mixed χ^2) with ten degrees of freedom. As a critical value for the control at the level of significance α = 5% we take $\chi^2_{0.95}(10)$ = 18.3070 which isn't less than the value of LR [Coelli et al., 2005]. Thus, the LR test leads us to *do not* reject the null hypothesis that the half-normal model isn't adequate (at the 5% level of significance), *which means that the Model* [C-D, Half-Normal, $N^+(0, \sigma_u^2)$] *is the statistically proposed model (at the 5% level of significance).*

Step 3: *We test hypotheses:* H_0: $\gamma = 0$ vs H_1: $\gamma > 0$.

We will additionally examine the presence of TE, after the application of this model. This will be achieved with the help of statistical function $Z=\hat{\gamma}/se(\hat{\gamma})$ which follows approximately, the distribution N(0,1). The control of the above hypothesis can also be performed with the LR criterion which approximately follows the distribution χ^2 with a degree of freedom, when the null hypothesis is valid: $\gamma = 0$. With the help of table A.4(in the Appendix), we find that LR =2.7533097. As a critical value for the control at the level of significance α = 5% we take $\chi^2_{0.1}(1)$ = 2.71 which is less than the value of LR. Thus, the LR test leads us to *reject* the null hypothesis that the half-normal model is adequate (at the 5% level of significance) [Coelli et al., 2005]. After all, the relatively high value of γ = 0.4788 means that 47.88% (i.e. 48%) of the change in the complex error is due to the U_i component of the inefficiency. This results, in this case, in the trend of identifying output-oriented TE, calculated by the DEA method, to that calculated by the SFA method. However, in the

Table 3
TE scores of DEA and SFA and the differences of their results

DMUs(1)	TE_{DEA}(2)	TE_{SFA}(3)	d(4)	DMUs(5)	TE_{DEA}(6)	TE_{SFA}(7)	d(8)
1	0.965	0.926	0.039	33	0.942	0.920	0.022
2	0.849	0.960	-0.111	34	0.978	0.878	0.1
3	0.833	0.946	-0.113	35	0.874	0.906	-0.032
4	1	0.944	0.056	36	1	0.914	0.086
5	1	0.907	0.093	37	1	0.945	0.055
6	0.924	0.918	0.006	38	1	0.958	0.042
7	0.849	0.902	-0.053	39	1	0.948	0.052
8	0.806	0.921	-0.115	40	0.951	0.890	0.061
9	0.814	0.919	-0.105	41	1	0.894	0.106
10	1	0.967	0.033	42	0.988	0.935	0.053
11	0.831	0.938	-0.107	43	1	0.937	0.063
12	0.878	0.922	-0.044	44	1	0.903	0.097
13	1	0.902	0.098	45	0.902	0.929	-0.027
14	1	0.895	0.105	46	0.95	0.909	0.041
15	0.971	0.944	0.027	47	1	0.942	0.058
16	0.961	0.970	-0.009	48	0.953	0.949	0.004
17	0.776	0.939	-0.163	49	0.876	0.920	-0.044
18	0.78	0.888	-0.108	50	0.932	0.920	0.012
19	0.837	0.927	-0.09	51	0.968	0.928	0.04
20	1	0.692	0.308	52	1	0.893	0.107
21	1	0.950	0.05	53	0.904	0.959	-0.055
22	0.845	0.926	-0.081	54	0.863	0.926	-0.063
23	0.989	0.960	0.029	55	0.911	0.921	-0.01
24	0.92	0.946	-0.026	56	1	0.951	0.049
25	0.889	0.944	-0.055	57	0.883	0.901	-0.018
26	0.8	0.907	-0.107	58	0.759	0.889	-0.13
27	0.902	0.918	-0.016	59	0.902	0.951	-0.049
28	0.877	0.902	-0.025	60	1	0.953	0.047
29	0.886	0.921	-0.035	61	0.939	0.921	0.018
30	0.814	0.919	-0.105	62	1	0.960	0.04
31	1	0.967	0.033	63	0.766	0.865	-0.099
32	0.813	0.938	-0.125	64	1	0.896	0.104

Source: Author's calculation

deviation of these two efficiencies participates for each unit i and the value v_i of the variable V_i of the statistical noise in a way that depends on the sign of v_i which does not remain constant.

Step 4: *We test hypotheses*: H_0: $\sigma_u^2 = 0$ vs H_1: $\sigma_u^2 > 0$.

Another test that examines the presence of the inefficiency variable in the complex error. Here again the statistical function $Z = \hat{\sigma}_u^2 / se(\hat{\sigma}_u^2)$ is used which approximately follows the normal distribution N (0, 1). From Table A.4, we find that LR = 2.7533097. The LR statistic follows approximately, the χ^2 distribution (more precisely a mixed χ^2) with a degree of freedom. As a critical value for the control at the level of significance α = 5% we take $\chi^2_{1-2*0.05}(1) = \chi^2_{0.9}(1) = 2.70554$ which is less than the value of LR, which means that with the criterion therefore the null hypothesis *is rejected* so the half-normal model is adequate (at the 5% level of significance) [Coelli et al., 2005]. Finally with the help of the statistic Z we check the statistical significance of the coefficients b_0, b_1, b_2, b_3, b_4. The values z_i, i =0, 2, 3 of the statistic Z for the coefficient from table A.4(in the Appendix), we find that are -4.9218639, 2.2004575, 9.0585556 their absolute value exceeds the critical value $z_{0.975} = 1.96$. As a consequence, the coefficients b_0, b_2, b_3 are statistically significant (at the 5% level of significance).

Therefore, from the above steps we conclude that the statistically preferred model for measuring efficiency with specific data, is a function of Cobb-Douglas production and inefficiency distribution in Half-Normal distribution. This model attributes the lack of efficiency, mainly to the inefficiency of the USSs and not to random factors. This can be interpreted to mean that the efficiency estimate achieved with this model is expected to be similar to the efficiencies calculated by the DEA method.

B. Measuring efficiency - Results

Measuring TE with DEA:

For data analysis we use the DEAP Version 2.1 software package [Coelli, 1996].

Measuring TE with SFA:

For data analysis we use the Frontier Version 4.1c software package [Coelli, 1996].

Furthermore, we get the results of SFA (Frontier Version 4.1c) and we calculate the differences (d) of results with DEA VRS(OO).

Regarding TE comparing its averages for the years 2020, 2021 and 2022 with the statistical method paired samples t-test, it turned out that there is no statistically significant difference between them (p> 0.05). For this reason, for further analysis, a variable will be used that will express the TE of the examined school units for all three years, which results from the average of TE. From the application of model 4 results are obtained which are presented in detail in the Table A.3 (in the Appendix). The next Table 3 presents the average values of the annual TEs of the examined school units for the years 2020, 2021 and 2022 with the two methods DEA VRS (OO) and SFA, with inputs X_1, X_2, X_3, X_4 and output Y_1. Columns (4) and (8) present the differences (d) of the results of the TE_{SFA} from TE_{DEA} of the school units.

The data in the Table 3 reveal: i. The proximity of the results, ii. With the SFA application no DMU has a TE equal to the unit. This finding confirms the existence of the value v_i of the random variable of the symmetric error V_i reported on the i-unit and expresses the general statistical noise. We observe that only 30 USSs (46.9%) have SFA efficiencies greater than their corresponding DEA VRS (OO).

It is also interesting to consider the existence of a correlation between SFA, DEA efficiencies and the results of their differences (d). Using IBM SPSS Statistics 23 we find that there is a statistically significant correlation (Spearman correlation) in the results of the applied methods, since p-value <0.05. Also, the correlation coefficient of the DEA VRS (OO) and SFA efficiencies is r = 0.193, so the relationship is that DEA VRS(OO) increases when the SFA increases, then the monotonic correlation is "weak positive". Therefore, the correlation coefficient of the DEA VRS (OO) and differences (d) is r = 0.932, so the relationship is that DEA VRS (OO) increases when the differences (d) increases, the monotonic correlation is "very strong positive", and the correlation coefficient of the SFA and the results of d is r =-0.136, so the relationship is that SFA decreases when the differences (d) increases, then the monotonic correlation is "very weak negative".

The Table 4 below shows the measures of descriptive statistics and the grouping of results with the two methods.

Table 4
Descriptive statistics of DEA and SFA scores

	DEA	SFA	d
Mean	0.923	0.922	0.00022
Median	0.941	0.924	0.009
Mode	1	0.921	-0.107
S.D.	0.076	0.379	0.083
C.V.[3]	8.2	41.1	37.727
Max	1	0.970	0.308
Min	0.759	0.692	-0.163
Range	0.241	0.278	0.471
Skewness	-0.555	-3.617	0.518
Kurtosis	-0.972	21.161	1.589

Source: Author's calculation

The findings, show that the average values of TE are at a satisfactory level (0.923 and 0.922), according to the international literature. This means that schools could produce outputs with fewer inputs by 7.7% and 7.8% respectively. The difference between the mean values of the TE techniques calculated by the two methods is very small (d = 0.00022). Similarly, for the median values, where the difference is d = 0.009. The relative variability (C.V.) is 0.082 (<0.10) and 41.1 (>0.10) respectively. That is, the data are a homogeneous sample only for the DEA VRS (OO) results. The Skewness is -0.555 <0 and - 3.617 <0 respectively, i.e. the distribution tail is shifted to the left, while the Kurtosis is - 0.972 (<3) (flat) and 21.161 (> 3) (thin) respectively. Additionally, the results of the measurements with DEA reveal that the thirty four USSs (53%) have higher than average efficiency, sixteen of the USSs (25%) that are characterized as relatively large, since they are larger than the average value of 163 students, with above-average efficiency, seventeen USSs (26.5%) with an efficiency higher than the average efficiency are based in the Capital of Regional Unity, six USSs (9.4%) with above-average efficiency if they set up prior to the year 2000. On the other hand, the results of the measurements with SFA reveal that thirty-two USSs (50%) have higher than average efficiency, eighteen (28%) that are characterized as relatively large with above-average efficiency, fifteen (23.4%) with an efficiency higher than the average efficiency are based in the Capital of Regional Unity, six USSs (9.4%) with above-average efficiency if they set up prior to the year 2000. We observe that only 30 USSs (46.9%) have SFA efficiencies greater than their corresponding DEA VRS (OO) and thirty four (53.1%) that have efficiencies less than their corresponding DEA VRS (OO). Another noteworthy point is that no unit has SFA efficiency equal to the unit. This means that in all units there is a problem of efficiency, i.e. the u_i is present. The Table 5 below shows the grouping of the results with the two methods.

Table 5
Grouping of TE_{DEA} and TE_{SFA} scores

	TE_{DEA}	TE_{SFA}
[0.600, 0.700)	0	1
[0.700, 0.800)	4	0
[0.800, 0.900)	19	10
[0.900, 1]	41	53

Figures 1 and 2 below show the frequency bar and the frequency curve of the TEDEA VRS (OO) and TESFA variables (Table 5), respectively.

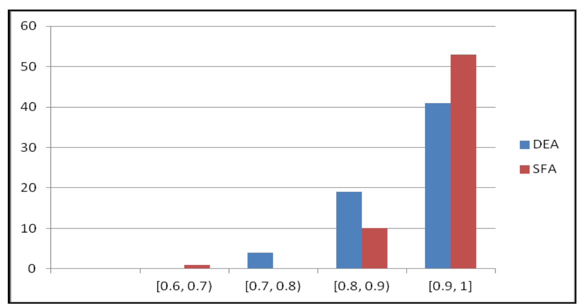

Fig. 1. TE_{DEA} and TE_{SFA} Frequency Chart

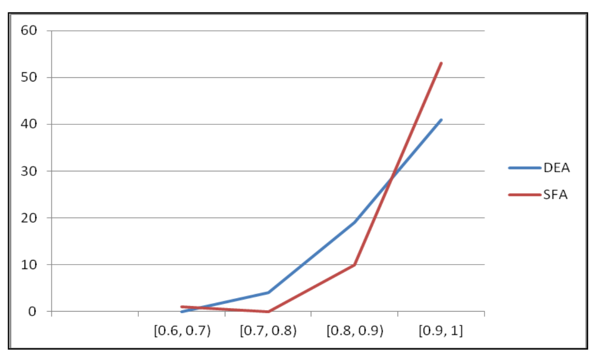

Fig. 2. TE_{DEA} and TE_{SFA} Frequency Curve

The Table 5 and figures 1 and 2, show us that there are only four (TE_{DEA}) and one (TE_{SFA}) observations at the total in the first two intervals [0.6, 0.7) and [0.7, 0.8) respectively. Almost all observations are in the intervals [0.8, 0.9) and [0.9, 1], for

[3] Coefficient of Variability

both TE_{DEA} and TE_{SFA}. It deserves attention the interval [0.9, 1], where observations for TE_{DEA} and TE_{SFA} are forty-one (64%) and fifty-three (82.8%) respectively. The relatively high value of $\gamma = 0.4788$ of C-D, Half-Normal (Table A.4 in the Appendix) means that 47.88% (i.e. 48%) of the change in the complex error is due to the U_i component of the inefficiency. With the SFA application no DMU has a TE equal to the unit. On the other hand, there are twenty-one (32.8%) observations for TE_{DEA} with value one (efficient units) in this interval.

C. A Short Discussion

The findings reveal that the upper secondary schools in Greece are small on the average, with a relatively small number of students per teacher and they also represent a large number of students per computer, compared to data from other countries (OECD, 2013, 2018). The technical efficiencies of the school units are, on average, close to the average levels of the findings of other tasks related to the same level of education (Ruggiero, J. and Vitaliano, F.D., 1999; Chakraborty et al., 2001).

The results of the technical efficiency of the school units, found by applying the two methods (DEA and SFA), show very small differences (average value d = 0.00022). It is noted that this is the first work in which the application of the SFA is done after selection by statistical methodology, of the Cobb-Douglas model, Half-Normal $N^{+}(0, \sigma_u^2)$, which presents the best adaptation to the empirical data, among four alternatives. The application of DEA gives slightly higher results (53.1%), compared to the corresponding ones given by the application of SFA (46.9%).

4. Concluding Remarks and Policy Recommendations

According to the economic literature, Marginalist (from 1870) and later Neoclassical economist gather their analytical interest in the microeconomic field. One of the key issues they are researching is the search for the best use of insufficient and competitively claimed resources from production systems. This also applies to educational units. The use of resources is evaluated by measuring efficiency and productivity. Efficiency is measured by the DEA and SFA methods. The paper mainly investigates empirically whether the measurements of TE units of higher secondary education with the methods DEA and SFA produce similar results. This study, with a statistical process, chooses among four alternatives the model with the best adaptation to the empirical data for the implementation of SFA. The empirical analysis shows that the results with the two methods are close. The average value of the difference in results is 0.00022 and the standard deviation is 0.083.

Taking everything into account, in the real field it can be considered that measurement with one method is sufficient.

Also, there is approximately equal number of USSs that are characterized as relatively large, with above-average efficiency. Moreover, there is approximately equal number of USSs based in the Capital of Regional Unity with above-average efficiency. Finally, there are equal number of USSs set up prior to the year 2000 with above-average efficiency. We observe that only 30 USSs (46.9%) have SFA efficiencies greater than their corresponding DEA VRS (OO) and thirty-four (53.1 %) that have efficiencies less than their corresponding DEA VRS (OO). And of course, the correlation coefficient of the DEA VRS (OO) and differences (d) is r = 0.932, so the relationship is "very strong positive".

From the above it follows as a policy proposal the establishment of an observatory whose mission will be the annual measurement of the efficiency of the school units and consequently suggestions to the administration and policy for improvement interventions in order to minimize the waste of resources.

According to the conclusions of the study, given its other advantages, the DEA is preferred.

List of Abbreviations

S.F.A.	Stochastic Frontier Analysis
D.E.A.	Data Envelopment Analysis
D.M.Us	Decision Making Units
USSs	Upper Secondary Schools
T.E.	Technical Efficiency
O.O.	Output Oriented
V.R.S.	Variable Returns to Scale
C-D	Cobb-Douglas
Tr	Transcendental Logarithmic
Vi	noise error term-symmetric (eg. normal distribution)
Ui	inefficiency error term-non-negative (eg.half-normal distribution)
I.O.	Input Oriented
AVG	average
S.D.	Standard Deviation
d	differences

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Appendix

Table A.1: The prices of the four inputs and one output, for the years 2020, 2021 & 2022 of the examined USSs

	INPUTS										OUTPUT		
	X_1			X_2			X_3			X_4	Y_1		
DMU	2020	2021	2022	2020	2021	2022	2020	2021	2022	2018, 2019, 2020	2018	2019	2020
1	304	293	297	30.92	27.5	26	805,428	705,240	672,104	41	90	57	85
2	278	292	287	25.08	25.33	23.67	677,506	701,431	685,645	36	64	41	100
3	160	144	157	15.83	15.83	14	371,621	402,077	394,324	24	40	32	44
4	215	242	213	20	20.92	20	480,067	546,500	566,365	10	52	46	59
5	248	252	228	30.17	30.75	30.67	787,852	810,061	832,979	15	50	51	68
6	127	118	129	15.17	16.17	17	400,783	402,778	474,519	22	31	38	29
7	86	88	68	11.83	11.17	13	321,892	317,790	372,431	13	15	20	17
8	76	79	86	7.08	10	10	281,527	261,268	279,881	23	9	11	19
9	115	105	109	12.92	14	14	356,477	372,671	394,901	18	29	30	24
10	48	42	39	8.08	7.08	8	155,229	161,343	184,620	5	12	12	7
11	92	85	93	19	11.08	12	295,193	268,983	297,907	12	31	15	16
12	79	79	77	12.33	9.92	9	180,757	220,482	212,837	21	19	22	16
13	157	157	171	13.75	19	18.17	461,682	484,539	482,939	31	47	49	39
14	162	161	160	11.83	15.67	15	296,174	368,507	370,362	26	25	27	25
15	207	188	174	21	21	20	514,214	545,336	520,752	26	55	55	67
16	143	127	87	15	16	16	338,228	405,046	401,935	28	35	44	31
17	120	117	114	12	12	12	286,705	312,551	316,246	19	28	36	28
18	69	78	78	12	11	11	260,939	263,507	268,877	10	12	21	18
19	79	85	89	11	11	11	275,068	290,051	260,294	16	17	19	22
20	428	426	426	36	37	37	872,360	882,873	914,644	43	98	136	128
21	442	436	401	30	30	30	932,105	761,661	776,852	44	103	106	129
22	250	280	292	23	23	23	563,166	586,542	596,542	29	57	51	73
23	381	381	352	27	27	27	727,308	734,771	731,818	32	117	80	102
24	353	327	303	28	26	25	760,734	700,257	673,260	38	97	99	89
25	277	271	259	22	22	18	575,088	592,240	502,286	34	53	74	63
26	167	179	181	17	16	16	407,794	401,919	420,802	26	31	40	34
27	290	307	304	23	23	23	569,923	579,062	593,597	25	69	61	90
28	212	248	255	18	18	18	416,768	431,626	449,202	18	40	49	58
29	171	174	176	18	17	17	810,497	458,995	474,709	15	48	43	46
30	169	158	159	18	17	16	388,051	386,966	363,166	16	24	33	28
31	165	148	133	14	14	14	337,338	343,330	361,510	17	51	48	50
32	179	175	176	18	18	18	757,792	411,703	426,228	25	25	30	50
33	167	164	175	13	12	12	304,286	311,911	309,496	16	39	36	46
34	68	58	68	11	11	10	280,672	288,800	264,719	9	20	16	23
35	58	57	63	8	7	7	225,718	196,883	224,679	12	14	18	12
36	66	56	56	5	5	4	130,126	119,890	113,336	11	11	5	11
37	195	192	184	17	18	184	471,058	472,332	457,435	22	52	60	56
38	338	332	288	26.45	26.93	288	773,309	815,928	823,271	27	95	98	108
39	212	217	200	18	17.45	200	559,590	550,427	517,662	25	63	63	64
40	278	281	294	24.06	24.44	294	700,065	397,296	751,945	36	87	88	73
41	328	356	362	28.67	27.9	362	823,392	832,142	802,175	28	90	83	125
42	247	278	283	23.53	22.43	283	638,413	623,921	603,755	34	66	89	77
43	288	291	292	19.45	18.95	292	554,315	556,520	559,406	20	75	72	84
44	42	47	45	8	7.98	45	240,568	238,759	215,178	14	10	6	15
45	80	94	78	7	6.98	78	205,011	210,453	216,124	16	18	20	20
46	91	86	66	8.21	7.48	66	225,460	219,945	237,465	30	20	26	16
47	113	81	65	9.39	9.97	65	270,914	286,061	292,305	9	38	25	15
48	168	145	136	18.61	17.95	136	485,879	479,295	463,041	34	42	39	34
49	76	85	66	9	8.98	66	228,265	249,309	240,618	17	13	18	15
50	133	126	117	10.5	9.97	117	253,579	255,713	274,758	13	23	23	36
51	50	58	66	6	5.98	66	159,227	163,904	173,183	16	7	10	19
52	78	94	86	10	9.97	86	261,768	276,548	276,725	15	19	18	16
53	96	89	89	11	10.97	89	268,547	277,450	277,656	16	18	15	23
54	70	75	73	9.61	8.47	73	261,768	238,615	218,662	19	16	12	20
55	92	81	77	9.06	5.98	77	239,535	235,038	246,298	12	17	20	18
56	201	225	204	19.07	18.95	204	480,648	496,978	478,056	23	44	42	37
57	54	53	52	7	6.98	52	166,827	179,217	175,814	13	10	10	11
58	169	177	173	18	17.95	173	490,076	455,187	530,820	35	38	35	35
59	177	170	188	20.83	19.25	188	555,542	580,657	554,765	30	46	47	57
60	55	45	47	5.08	5.08	47	110,629	117,223	146,827	14	12	5	14
61	52	54	49	7.83	7.83	49	192,026	197,747	203,966	15	5	10	10
62	176	169	148	13.92	13.17	148	306,062	299,318	409,636	16	46	50	58
63	61	63	69	9.75	9.75	69	261,027	269,016	221,262	18	11	12	12
64	34	35	34	5.08	5.08	34	125,629	129,716	153,400	11	6	8	10

Source: Author's calculation

Table A.2: DEA VRSTE (OO) scores for the years 2018, 2019 & 2020

DMU	2018	2019	2020	DMU	2018	2019	2020
1	1	0.996	0.9	33	0.912	0.913	1
2	0.822	0.739	0.985	34	1	0.934	1
3	0.858	0.835	0.807	35	0.914	1	0.708
4	1	1	1	36	1	1	1
5	1	1	1	37	1	1	1
6	0.87	1	0.901	38	1	1	1
7	0.604	1	0.942	39	1	1	1
8	0.804	0.863	0.752	40	1	1	0.852
9	0.84	0.872	0.731	41	1	1	1
10	1	1	1	42	0.964	1	1
11	1	0.708	0.785	43	1	1	1
12	0.857	0.912	0.864	44	1	1	1
13	1	1	1	45	0.841	0.918	0.947
14	1	1	1	46	0.88	1	0.97
15	1	0.914	1	47	1	1	1
16	0.883	1	1	48	0.858	1	1
17	0.711	0.945	0.672	49	0.763	1	0.866
18	0.775	0.864	0.702	50	0.872	0.923	1
19	0.81	0.833	0.868	51	0.904	1	1
20	1	1	1	52	1	1	1
21	1	1	1	53	0.88	0.832	1
22	0.951	0.767	0.818	54	0.814	0.774	1
23	1	1	0.968	55	0.734	1	1
24	0.896	1	0.865	56	1	1	1
25	0.855	1	0.812	57	0.775	0.968	0.907
26	0.766	1	0.635	58	0.802	0.767	0.708
27	0.829	0.877	1	59	0.95	0.96	0.795
28	0.688	1	0.944	60	1	1	1
29	1	0.867	0.791	61	1	1	0.816
30	0.797	0.888	0.756	62	1	1	1
31	1	1	1	63	0.72	0.749	0.83
32	0.816	0.8	0.824	64	1	1	1

Table A.3: SFA scores for the years 2018, 2019 & 2020

DMU	2018	2019	2020	DMU	2018	2019	2020
1	0.920	0.921	0.921	33	0.919	0.920	0.920
2	0.927	0.927	0.928	34	0.878	0.878	0.879
3	0.952	0.952	0.952	35	0.905	0.906	0.906
4	0.944	0.944	0.946	36	0.913	0.914	0.914
5	0.936	0.937	0.937	37	0.944	0.945	0.945
6	0.903	0.903	0.904	38	0.958	0.958	0.958
7	0.914	0.914	0.914	39	0.948	0.948	0.949
8	0.937	0.938	0.938	40	0.890	0.890	0.891
9	0.939	0.940	0.940	41	0.894	0.894	0.895
10	0.909	0.910	0.910	42	0.935	0.935	0.935
11	0.915	0.916	0.916	43	0.937	0.937	0.937
12	0.921	0.922	0.922	44	0.903	0.903	0.904
13	0.901	0.902	0.902	45	0.929	0.929	0.929
14	0.894	0.895	0.895	46	0.908	0.909	0.909
15	0.943	0.944	0.944	47	0.941	0.942	0.942
16	0.970	0.970	0.971	48	0.949	0.949	0.949
17	0.939	0.939	0.939	49	0.920	0.920	0.921
18	0.888	0.888	0.888	50	0.919	0.920	0.920
19	0.927	0.927	0.927	51	0.928	0.928	0.929
20	0.690	0.692	0.694	52	0.892	0.893	0.893
21	0.950	0.950	0.950	53	0.959	0.959	0.959
22	0.925	0.926	0.926	54	0.926	0.926	0.927
23	0.960	0.960	0.960	55	0.921	0.921	0.922
24	0.946	0.946	0.946	56	0.950	0.951	0.951
25	0.943	0.944	0.944	57	0.900	0.901	0.901
26	0.906	0.907	0.907	58	0.889	0.889	0.890
27	0.918	0.918	0.919	59	0.951	0.951	0.951
28	0.902	0.902	0.903	60	0.953	0.953	0.953
29	0.920	0.921	0.921	61	0.920	0.921	0.921
30	0.918	0.919	0.919	62	0.960	0.960	0.960
31	0.967	0.967	0.967	63	0.864	0.865	0.866
32	0.938	0.938	0.938	64	0.895	0.896	0.896

Table A.4: Output from the program Frontier (Version 4.1c)

The final mle estimates are:			
Log likelihood function =-0.53566976E+01			
	coefficient	**standard-error**	**t-ratio**
b_0	-0.73302655E+00	0.14893272E+00	-0.49218639E+01
b_1	-0.78670171E-09	0.53264704E-10	-0.14769663E+02
b_2	0.89700416E+00	0.40764438E-01	0.22004575E+02
b_3	0.68499169E-09	0.75618202E-10	0.90585556E+01
b_4	0.73210776E-01	0.49746401E-01	0.14716798E+01
sigma^2	0.10709800E+00	0.70610238E-01	0.15167489E+01
gamma	0.47883731E+00	0.35871653E+00	0.13348627E+01
mu	-0.45291288E+00	0.88978412E+00	-0.50901434E+00
eta	0.63313080E-02	0.18110681E+00	0.34958972E-01
log likelihood function (LnL_R)= - 0.39800427E+01			

Source: Author's calculation

The final mle estimates are:			
	coefficient	**standard-error**	**t-ratio**
b_0	-0.71083219E+00	0.16293741E+00	-0.43626088E+01
b_1	-0.78239792E-09	0.53247870E-10	-0.14693506E+02
b_2	0.89699609E+00	0.44017939E-01	0.20377967E+02
b_3	0.69198057E-09	0.78020051E-10	0.88692659E+01
b_4	0.69406840E-01	0.50564633E-01	0.13726361E+01
sigma^2	0.71651991E-01	0.13646193E-01	0.52506944E+01
gamma	0.22005395E+00	0.16745865E+00	0.13140793E+01
mu	is restricted to be zero		
eta	0.25437097E-01	0.20407078E+00	0.12464841E+00
log likelihood function (LnL_U)= - 0.42632236E+01			

Source: Author's calculation

Table A.6: Output from the program Frontier (Version 4.1c)

The final mle estimates are:			
	coefficient	**standard-error**	**t-ratio**
b_0	-0.73368206E-01	0.91941250E+00	-0.79799009E-01
b_1	-0.80464141E-09	0.47831722E-10	-0.16822338E+02
b_2	0.19657569E+00	0.10381917E+00	0.18934431E+01
b_3	0.14456097E-09	0.10292094E-09	0.14045827E+01
b_4	0.67846794E-01	0.11021497E+00	-0.61558601E+00
b_{11}	0.13675894E-09	0.10168348E-09	0.13449475E+01
b_{22}	-0.64257336E-01	0.11964895E+00	-0.53704888E+00
b_{33}	0.44422342E-09	0.10615339E-08	0.41847313E+00
b_{44}	0.47028148E-01	0.51908009E-01	0.90599021E+00
b_{12}	0.20244786E-10	0.69913410E-10	0.28956942E+00
b_{13}	0.77780477E-01	0.13131058E-01	0.59233973E+01
b_{14}	0.50824563E-09	0.56207739E-09	0.90422714E+00
b_{23}	-0.39639957E-01	0.34632641E-01	-0.11445837E+01
b_{24}	0.13443231E-09	0.59255367E-10	0.22686942E+01
b_{34}	0.98479131E-02	0.70408798E-02	0.13986765E+01
sigma^2	0.11829251E+00	0.46478279E-01	0.25451137E+01
gamma	0.67373543E+00	0.12439499E+00	0.54160978E+01
mu	-0.56461616E+01	0.46700524E+00	-0.12090146E+01
eta	-0.13236580E+00	0.12731003E+00	-0.10397123E+01
log likelihood function (LnL_R)= 0.25274026E+02			

Source: Author's calculation

The final mle estimates are:			
	coefficient	standard-error	t-ratio
b_0	-0.33644811E+00	0.95640784E+00	-0.35178310E+00
b_1	-0.79572897E-09	0.47377668E-10	-0.16795444E+02
b_2	0.19480299E+00	0.10589684E+00	0.18395543E+01
b_3	0.14613436E-09	0.10329112E-09	0.14147814E+01
b_4	0.84024899E-01	0.11424771E+00	0.73546246E+00
b_{11}	0.14624845E-09	0.10342105E-09	0.14141072E+01
b_{22}	-0.34808615E-01	0.12222234E+00	-0.28479748E+00
b_{33}	0.68631513E-09	0.10848762E-08	0.63262069E+00
b_{44}	0.42685556E-01	0.52741835E-01	0.80933014E+00
b_{12}	0.12426114E-10	0.70781500E-10	0.17555596E+00
b_{13}	0.78785172E-01	0.13264757E-01	0.59394359E+01
b_{14}	0.56769536E-09	0.56346551E-09	0.10075068E+01
b_{23}	-0.49354720E-01	0.35555894E-01	-0.13880883E+01
b_{24}	0.13412860E-09	0.59651984E-10	0.22485187E+01
b_{34}	0.93903784E-02	0.70802999E-02	0.13262685E+01
sigma^2	0.62665234E-01	0.13063784E-01	0.47968670E+01
gamma	0.37188015E+00	0.15291572E+00	0.24319287E+01
mu	is restricted to be zero		
eta	-0.10654364E+00	0.15477321E+00	-0.68838556E+00
log likelihood function (LnL_U)= 0.24439548E+02			

Source: Author's calculation

Table A.8: Environmental Variables of USSs

DMU	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8
1	1	1	0	10.64	21,670	0.965	0.926	0.039
2	1	1	0	11.58	21,670	0.849	0.960	-0.111
3	0	0	0	10.14	21,670	0.833	0.946	-0.113
4	1	1	0	10.99	21,670	1	0.944	0.056
5	1	1	0	7.95	21,670	1	0.907	0.093
6	0	1	1	7.75	21,670	0.924	0.918	0.006
7	0	0	1	6.79	21,670	0.849	0.902	-0.053
8	0	0	0	9.08	21,670	0.806	0.921	-0.115
9	0	0	0	8.06	21,670	0.814	0.919	-0.105
10	0	0	0	5.58	21,670	1	0.967	0.033
11	0	0	0	6.75	21,670	0.831	0.938	-0.107
12	0	0	0	7.64	21,670	0.878	0.922	-0.044
13	0	0	1	9.70	21,670	1	0.902	0.098
14	0	0	0	11.55	21,670	1	0.895	0.105
15	1	1	0	8.94	13,698	0.971	0.944	0.027
16	0	1	0	7.64	13,698	0.961	0.970	-0.009
17	0	0	0	9.75	13,698	0.776	0.939	-0.163
18	0	0	1	6.64	13,698	0.78	0.888	-0.108
19	0	0	0	7.67	13,698	0.837	0.927	-0.09
20	1	1	0	11.64	13,698	1	0.692	0.308
21	1	1	0	14.21	13,698	1	0.950	0.05
22	1	1	1	11.91	13,698	0.845	0.926	-0.081
23	1	1	0	13.75	13,698	0.989	0.960	0.029
24	1	0	0	12.43	13,698	0.92	0.946	-0.026
25	1	0	0	13.10	13,698	0.889	0.944	-0.055
26	1	0	1	10.78	13,698	0.8	0.907	-0.107
27	1	0	0	13.06	13,698	0.902	0.918	-0.016
28	1	0	0	13.24	13,698	0.877	0.902	-0.025
29	1	0	0	10.03	13,698	0.886	0.921	-0.035
30	0	0	0	9.54	13,698	0.814	0.919	-0.105
31	0	0	1	10.62	13,698	1	0.967	0.033
32	1	0	0	9.81	13,698	0.813	0.938	-0.125
33	1	0	0	13.70	13,698	0.942	0.920	0.022
34	0	0	0	6.09	13,698	0.978	0.878	0.1
35	0	0	0	8.13	13,698	0.874	0.906	-0.032
36	0	0	0	12.80	13,698	1	0.914	0.086
37	1	1	0	11.21	10,197	1	0.945	0.055
38	1	1	0	11.93	14,113	1	0.958	0.042
39	1	1	1	12.12	14,113	1	0.948	0.052
40	1	1	0	11.62	14,113	0.951	0.890	0.061
41	1	1	1	17.20	14,113	1	0.894	0.106
42	1	1	1	11.44	14,113	0.988	0.935	0.053
43	1	1	0	12.85	14,113	1	0.937	0.063
44	0	0	0	5.59	14,113	1	0.903	0.097
45	0	0	1	12.02	14,113	0.902	0.929	-0.027
46	0	0	0	10.28	14,113	0.95	0.909	0.041
47	0	0	0	8.89	14,113	1	0.942	0.058
48	0	0	0	8.23	14,113	0.953	0.949	0.004
49	0	0	0	8.42	14,113	0.876	0.920	-0.044
50	0	0	0	12.35	14,113	0.932	0.920	0.012
51	0	0	0	9.69	14,113	0.968	0.928	0.04
52	0	0	0	8.62	14,113	1	0.893	0.107
53	0	0	0	8.32	14,113	0.904	0.959	-0.055
54	0	0	0	8.43	14,113	0.863	0.926	-0.063
55	0	0	0	10.76	14,113	0.911	0.921	-0.01
56	1	0	0	11.06	14,113	1	0.951	0.049
57	0	0	0	7.59	14,113	0.883	0.901	-0.018
58	1	0	1	9.63	10,734	0.759	0.889	-0.13
59	1	1	1	8.47	10,734	0.902	0.951	-0.049
60	0	0	0	9.22	10,734	1	0.953	0.047
61	0	0	0	6.55	10,734	0.939	0.921	0.018
62	1	1	0	11.16	10,734	1	0.960	0.04
63	0	0	0	7.11	10,734	0.766	0.865	-0.099
64	0	0	0	6.39	10,734	1	0.896	0.104

Source: Author’s calculation