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Application of Graph Theory in Mechanical Engineering Design for Kinematics Chain Mechanism

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***Abstract*: The theory of machines and mechanisms is an applied science that is used to understand the relationships between the geometry and motions of the parts of a machine or mechanism and the forces that produce these motions. The subject can be divided into three parts namely (i) mechanisms and kinematics of mechanisms. (ii) methods of designing mechanisms and (iii) study of kinetics, the time- varying forces in machines. Among these tools are software packages like Mathematics, MATLAB, TK@ Solver, and Math@ CAD. These packages have the advantage of allowing the user to document and save completed work in a highly detailed fashion. In the present work, the software MATLAB has been used as a computational tool.**

***Keywords*: Theory of machine, MATLAB, Kinematics.**

1. Introduction

Design is the creation of synthesized solutions in the form of products or systems that satisfy customer's requirements. When we are given a design problem, we try to make the best use of our knowledge and the available information to understand the problem and generate as many feasible solutions as possible.

The theory of machines and mechanisms is an applied science that is used to understand the relationships between the geometry and motions of the parts of a machine or mechanism and the forces that produce these motions for an optimal design.

Thus, the knowledge of this is very essential for an engineer in designing the various parts of the machine.

A. Basic Terminologies

Link/Element: Each part of a machine that moves relative to another part is known as a Kinematic link.

Kinematic Pair: The two links of a machine when in contact with each other and if the relative motion between them is completely constrained then it is called a kinematic pair.

Kinematic chain: A kinematic chain is an assembly of links connected by joints, which allow relative motion between them. In other words, if few links are connected, and there is a relative motion, it's considered a Kinematic Chain.

Degree of freedom: A link is supposed to have n degrees of freedom if it has *n* independent variables associated with its position in the plane.

An equation to determine the degrees of freedom of a mechanism can be written as:

$$F = 3(n-1) - 2J - h$$

Where,
n=Number of links j=Number of lower pair
h= number of higher pair

Grashof's Law: According to Grashof's law a four-bar linkage framework, the sum of the shortest and longest link of a planar quadrilateral linkage is less than or equal to the sum of the remaining two links, then the shortest link can rotate fully concerning an adjoining link. Let us consider a four-bar-linkage system which represents the smallest link by S, the longest link by L, and the & other two links by P and Q. In the case of Grashof's Law condition is satisfied i.e. $S+L < P+Q$, Then depending on whether the shortest link 'S' is associated with the ground by one end, two ends, or no end there are 3 possible mechanisms.

1. Double Crank Mechanism
2. Double Rocker mechanism
3. Crank and Rocker Mechanism

B. Literature Review

We did different paper reviews to collect more ideas and to identify the problem. Its primary purpose is to understand the latest advancements in research within these fields, identify existing gaps in knowledge, and prevent redundancy in subsequent work.

A comprehensive Evaluation of various studies is shown in table 1.

C. Research Gap

After reading the reviews and different algorithms/methods for the solution to the problem we will try to optimise further those solutions.

D. Problem Statement

In this project a new, reliable, and more efficient way to check isomorphism and determine their mechanism for planar

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Table 1
Literature review

S.No.	Author & Journal	Title	Brief of Paper
1	Ashok Dargar, Ali Hasan and R.A Khan. December 2009 International Journal of Mechanical and Materials Engineering 4(3).	Identification of isomorphism among kinematic chains and inversions using link adjacency values.	Efficient and reliable method to detect isomorphism between kinematic chain and among inversions of a given chain.
2	Sayeed Ahamad, Aas Mohammad and Jiyaul Mustafa. March 2016	Identification of Isomorphism of Kinematic Chains Using Different Kinematic Pairs.	Identification of isomorphism among the kinematic chains using different kinematic pairs, by considering the examples of pump mechanisms for a manual hand pump and nosewheel assembly for a small aircraft.
3	Luchuan Yu, Chenxu Cai, Jianhua Zhang, Qinhe Zhang. November 2021Soft Computing 25(4).	A simple and efficient method for isomorphism identification of planar kinematic chains.	Based on the adjacency matrix and loop theory, a new method is proposed in this paper to identify the isomorphic kinematic chains.
4	Rashmi Arora, S.P Nigam. 2008/8/19 ME Thesis. Thapar University, Patiala (Punjab)–India	A Comparative Study of Various Methods for Identification of Isomorphism in Kinematic Chains.	isomorphism in kinematic chains, crucial for efficient mechanism analysis. Various methods were compared using Watt and Stephenson Chains, aiding method selection.
5	Sankalp Verma, P B Deshmukh.	Identification Of Distinct Inversions In A Kinematic Chain Using Link-Joint- Loop Adjacency	This paper approaches a new method based on matrix composition to identify the accurate number of distinct mechanisms that can be obtained in a particular kinematic chain.

kinematic chains.

The method is explained with the help of examples of kinematic chains with the distinct mechanisms of kinematic chains of 6 and 8 links.

This approach will be helpful for the new researchers and designers and will reduce unnecessary effort.

2. Isomorphism of Kinematic Chain of 6-Link

A. Introduction

For the study of isomorphism and distinct mechanism of planar mechanism of kinematic Chains of six links and seven joints having a single degree of freedom [JJ] matrix is defined based upon the connectivity of join through the link. Two chains are isomorphic if their sum absolute eigenvalues are the same.

B. Objectives

To develop the J-J matrix, to identify isomorphism and distinct mechanism of six link and seven joints, having single DOF, Kinematic chains, and Mechanisms.

C. The Joint-Joint [JJ] Matrix

This matrix is based upon the connectivity of the joints through the links and is defined, as a square symmetric matrix of size n x n. where n is the number of joints in a kinematic chain.

Note: The joints of a chain are assigned positive integers 1, 2, 3..., and n as their names while the small English letters a, b, c, etc. are used for labeling the links of a kinematic chain.

1) Degree of link d (li)

The degree of link represents the type of link like binary, ternary, quaternary, etc. Let the degree of ith link in a kinematic chain be designed as d (li).

d (1i)=2, for the binary link,
d (1i)=3, for ternary link,
d (1i)= 4, for the quaternary link,
d (1i)=k, for k-nary link.

[JJM] = {Lij} n x n Where,

Lij = Degree of link between i and joints those are directly connected=0, if joint i is not directly connected to joint j all the diagonal elements Lii=0

D. Summary of the Procedure for Detection of Isomorphism Among the Kinematic Chains

The following systematic steps are followed to detect isomorphism among the kinematic chains using the sum of eigenvalues and max. eigenvalues.

Step-1:

Sketch the kinematic chains and give the number to the joints in an arbitrary manner from 1 to n.

Step-2:

Develop the Graph of the Kinematic Chain by proper placements of the link so that the links do not cross each other.

Step-3:

Develop the [JJ] matrix from the graph as: [JJ] = {Lij}

Where,

Lij = Degree of link between i and joints those are directly connected =0, if joint i is not directly connected to joint j all the diagonal elements Lii=0

Step-4:

Determine the sum of the absolute eigenvalues and maximum absolute eigenvalues by fixing each link respectively. When the link is fixed then the particular diagonal element corresponding to that link will be taken as 1. If they are isomorphic kinematic chains then their sum of the absolute eigenvalues is otherwise treated as a distinct mechanism.

For Example: For the six link Kinematic chain shown here the different matrices that will be formed by fixing each link are,

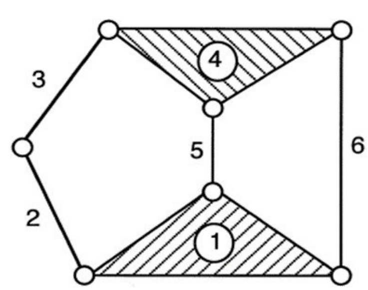


Fig. 1.

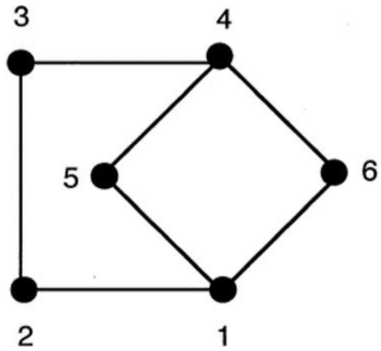


Fig. 2. Corresponding graph

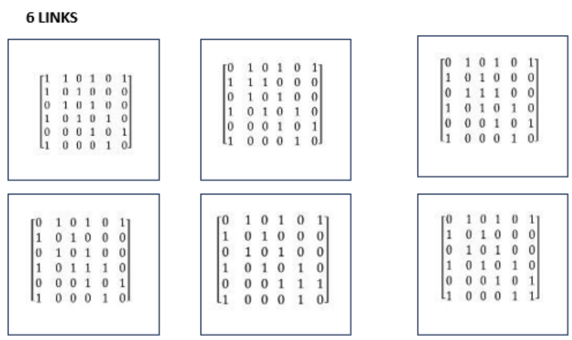


Fig. 3.

Then we will determine the Absolute Sum of Eigen Values using MATLAB as shown in the figure below.

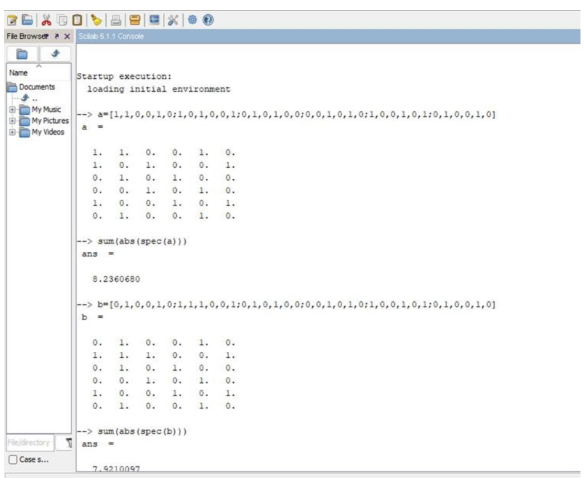


Fig. 4.

For the Kinematic chain,

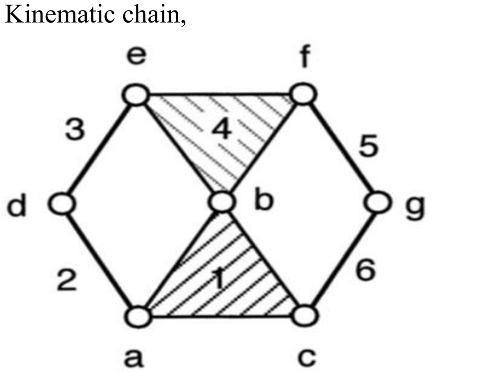


Fig. 5.

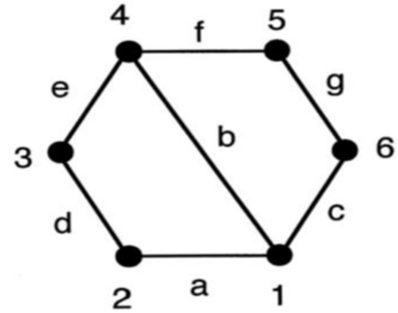


Fig. 6. Corresponding graph

The matrices formed are by fixing each link,

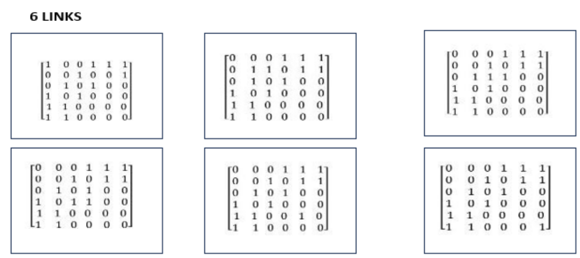


Fig. 7.

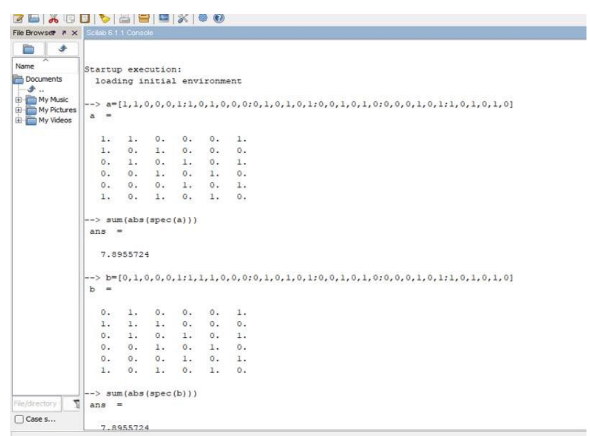


Fig. 8. Using MATLAB

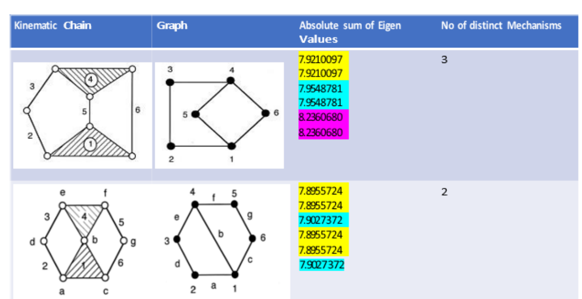


Fig. 9. 6 link mechanisms

E. Conclusion

There are 2 different Kinematic chains of 6 links and 7 joints and we have developed different matrices on fixing different links of each kinematic chain and there are around 12 matrices.

And we are mainly focused on the adjacency matrix and the corresponding eigenvalues. We then calculated the absolute sum of the eigenvalues and compared the mechanisms.

Thus, we got around 5 different mechanisms for 6 link kinematic chains.

3. Isomorphism of Kinematic Chain of 8-Link

A. Introduction

For the study of isomorphism and distinct mechanism of planar mechanism of kinematic Chains of eight link and ten joints having a single degree of freedom [JJ] matrix is defined based upon the connectivity of join through the link. Two chains are isomorphic if their sum absolute eigenvalues are the same.

B. Objectives

To develop a J-J matrix, to identify isomorphism and distinct mechanism of eight link and ten joints, having single DOF, Kinematic chains, and Mechanisms.

C. The Joint- Joit [JJ] Matrix

This matrix is based upon the connectivity of the joints through the links and is defined, as a square symmetric matrix of size n x n. where n is the number of joints in a kinematic chain.

Note: The joints of a chain are assigned positive integers 1, 2, 3...,n as their names while the small English letters a, b, c, etc. are used for labeling the links of a kinematic chain.

Lij= chains then their sum of the absolute eigenvalues is otherwise treated as a distinct mechanism.

= $Degree\ of\ link\ between\ i\ and\ joints\ those\ are\ directly\ connect$ similarly as shown in the previous section we will develop,

= 0, $if\ joint\ i\ is\ not\ directly\ connected\ to\ joint\ j$ all the diagonal elements Lii=0

D. Summary of the Procedure for Detection of Isomorphism among the Kinematic Chains

The following systematic steps are followed to detect isomorphism among the kinematic chains using the sum of eigenvalues and max. eigenvalues.

Step-1:

Sketch the kinematic chains and give the number of joints in the arbitrary manner from 1 to n.

Step-2:

Develop the Graph of the Kinematic Chain by proper placements of the link so that the links do not cross each other.

Matrices for Kinematic chains of 8 links and compare their sum of eigenvalues.

The detailed study is shown in the figures given below,

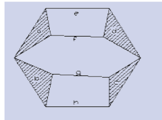
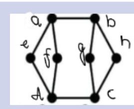
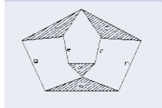
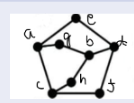
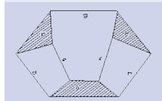
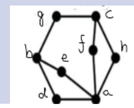
Kinematic	Graph	Absolute sum of Eigen values	No of distinct
		10.880769 10.88076 9 10.88076 9 10.41378 5 10.413785	2
		11.63485 11.411003 11.411003 11.634851 11.411003 11.411003 11.634851 11.634851	2
		10.859636 10.377966 [illegible] 10.37796 10.859636 10.365542 10.859636 10.859636	4

Fig. 10.

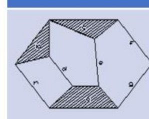
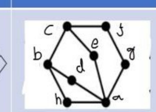
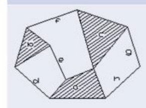
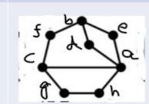
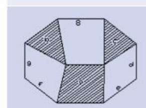
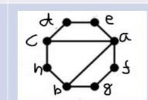
Kinematic Chain	Graph	Absolute sum of Eigen values	No of distinct mechanism
		11.372455 10.986054 10.995974 10.993836 11.002915 10.973912 11.372455 11.224615	7
		10.376952 10.363436 10.408277 10.236609 10.387309 10.642563 10.382410 10.642563	7
		10.474646 10.524737 10.438883 10.227429 10.438883 10.524737 10.474646 10.483729	5

Fig. 11.

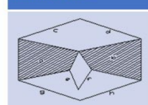
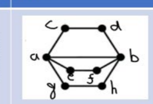
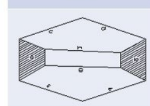
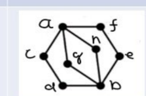
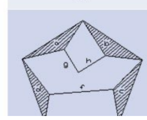
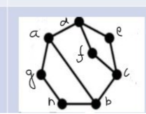
Kinematic Chain	Graph	Absolute sum of Eigen values	No of distinct mechanism
		11.142637 11.142637 11.102748 11.142637 11.142637 11.102748 11.142637 11.142637	2
		10.956671 10.956671 10.920072 10.956671 10.956671 10.920072 11.354819 11.354819	3
		10.648444 10.185466 10.418178 10.377941 10.377941 10.418178 10.185466 10.648444	4

Fig. 12.

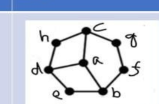
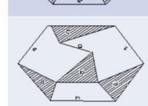
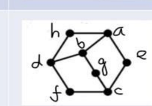
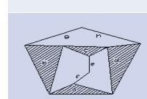
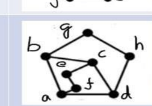
Kinematic Chain	Graph	Absolute sum of Eigen values	No of distinct mechanism
		10.871183 10.833531 10.888007 10.813809 10.888007 10.833531 10.871183 11.163688	5
		11.374177 11.426694 11.384688 11.426694 11.374177 11.182495 11.446142 11.483391	6
		11.483920 11.478863 11.478863 11.483920 11.483920 11.483920 11.478863 11.478863	2

Fig. 13.

There are 16 different Kinematic chains of 8 links and 10 joints and we have developed different matrices on fixing different links of each kinematic chain and there are around 128 matrices.

And we are mainly focused on the adjacency matrix and the corresponding eigen values. We then calculated the absolute sum of the eigen values and compared the mechanisms.

Thus, we got around 71 different mechanisms for 8 link kinematic chains.

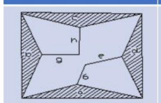
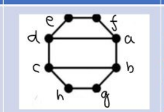
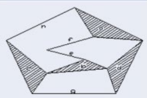
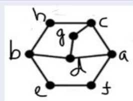
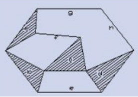
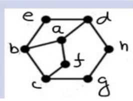
Kinematic Chain	Graph	Absolute sum of Eigen values	No of distinct mechanism
		10.737460 10.737460 10.691561 10.691561 10.737460 10.737460 10.691561 10.691561	2
		11.175117 11.171975 11.196021 11.208415 11.266729 11.199294 11.298080 11.376981	8
		11.305770 11.305770 11.283341 11.364588 11.329599 11.283341 11.364588 11.329599	4

Fig. 14.

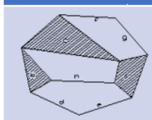
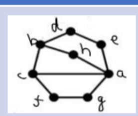
Kinematic Chain	Graph	Absolute sum of Eigen values	No of distinct mechanism
		10.916419 10.901724 10.943724 10.919285 10.996615 10.982020 10.927038 11.026079	8

Fig. 15.

E. Conclusion

There are 16 different Kinematic chains of 8 links and 10 joints and we have developed different matrices on fixing different links of each kinematic chain and there are around 128 matrices.

And we are mainly focused on the adjacency matrix and the corresponding eigenvalues. We then calculated the absolute sum of the eigenvalues and compared the mechanisms.

Thus, we got around 71 different mechanisms for 8 link kinematic chains.

4. Conclusion

In the present work, a simple, efficient, and reliable method is used for the determination of Isomorphism and detection of the same. A matrix called the Joint-Joint [J] matrix has been studied here. The same [JJ] matrix has been used for the determination of the Distinct Mechanisms from the given Kinematic Chain. With the help of computational tools (here MATLAB) we develop the [JJ] matrix and easily determine their absolute eigenvalues and their maximum absolute eigenvalues.

There are 2 different Kinematic chains of 6 links and 7 joints and we have developed different matrices on fixing different links of each kinematic chain and there are around 12 matrices. Thus, we got around 5 different mechanisms for 6 link kinematic chains.

There are 16 different Kinematic chains of 8 links and 10 joints and we have developed different matrices on fixing different links of each kinematic chain and there are around 128 matrices. Thus, we got around 71 different mechanisms for 8 link kinematic chains.

5. Future Work

The following work can be placed in the column of scope of future work.

1. We will deduce or determine some new, simple, and reliable method to identify the isomorphism of kinematic chains and their Distinct Mechanisms.
2. Generate a method for the skeleton of Kinematic Chains. And we will determine the best kinematic chain.
3. The kinematic chain may be represented by any other method new matrix using the concept of different kinematic pairs.
4. The kinematic chain may be represented by any other method rather than a matrix.
5. This work may be extended using the concept of friction, wear, stiffness, or any other parameter.

References

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