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Mathematical Approach for Enhancing Audio Signal Quality: Theory, Insights, and Applications

Moin Tariq[1*], Muhammad Irfan Haider Khan[2]
[1,2]Research Scholar, Department of Mathematics & Statistics, International Islamic University, Islamabad, Pakistan

***Abstract*: In real-life audio signal transmission, noise significantly diminishes signal quality. This paper explains a technique to mitigate noise in audio signals. The proposed technique leverages the Fast Fourier Transform (FFT) to filter noisy signals, with a focus on White noise (AWGN) as the noise source. The underlying mathematical theory is comprehensively explored, accompanied by practical insights derived from a selected sample frequency. The significance of noise reduction in enhancing audio signal quality within communication systems is underscored. This study contributes to the understanding of the mathematics behind noise reduction techniques and their application in real-world scenarios.**

***Keywords*: audio processing, de-noising, fourier series and transforms, sampling frequency transform, inverse fourier transform, white noise.**

1. Introduction

In the realm of telecommunications, noise, and distortion stand as key limitations to data transmission capacity and signal measurement accuracy [1]. As such, the modeling and mitigation of these factors form pivotal considerations in both theoretical and practical aspects of communications and signal processing. This becomes particularly pronounced in applications such as mobile communication, speech recognition, image and signal processing, signal analysis in medical, radar, sonar, and scenarios where wanted signals contend with noise and distortion [2].

This paper delves into a specific technique employed for noise reduction, offering a mathematical interpretation that serves as its foundation [3]. The development of the mathematical theory underpinning this noise reduction technique, signal representation, the Fourier series, and the Fourier Transform, providing theoretical insights. Then the method is applied through MATLAB scripting, generating plots of spectral profiles for various functions under examination.

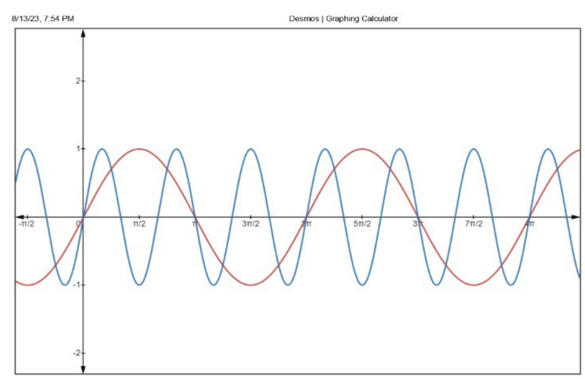
These efforts culminate in Conclusions drawn from a comparison between the noisy signals and their de-noised counterparts, where the wanted signals cannot be separated from noise and distortion. The method, has been then used by the authors to de-noise a real-time audio signal under consideration.

*Corresponding author: sci.mointariq@gmail.com

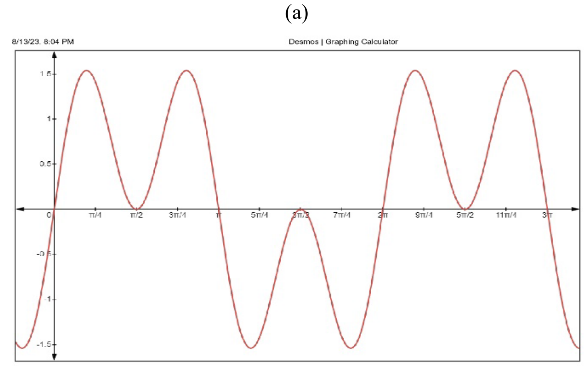
2. Background

A. Mathematical Representation and Analysis of Waves using Fourier Series

Mathematically, any waveform characterized by a specific frequency and amplitude could be stated in terms of sine or/and cosine functions [4]. These fundamental trigonometric functions inherently create smooth curves, capturing the cyclic nature of waves [5]. However, in practical analysis and decomposition, the waveform's representation can be transformed to exhibit sharp changes. This transformation provides a more insightful perspective for in-depth analysis. The concept is visually depicted in the following Figures (Fig. 1 & 2):



(a)



(b)

Fig. 1. a) Individual periodic functions, b) Sum of periodic functions

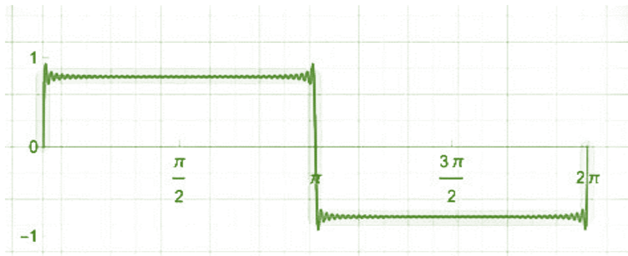


Fig. 2. Plot of the sum of finitely many functions of different frequencies and amplitude

Fourier series, a cornerstone of signal analysis, emerges as a crucial tool in this context. It dissects a complex function into its constituent waveforms, each with distinct frequencies and amplitudes, which when combined, reconstruct the original function [6]. This decomposition and reconstitution using the Fourier series allow for a granular examination of the individual frequency components comprising a complex signal. By leveraging the Fourier transform, these constituent frequencies can be isolated and analyzed, enabling the extraction of meaningful information and the removal of undesired noise.

Ultimately, the Fourier series and transform empower researchers and practitioners to untangle the convoluted interplay of frequencies within a waveform [7]. This process is pivotal in signal processing, communications, and diverse scientific disciplines, driving advancements and deeper insights into the nature of signals.

In summary, the utilization of the Fourier series and transform stands as a profound methodological pillar, enhancing our ability to comprehend and manipulate complex waveforms for a myriad of applications.

B. Mathematical Foundations of Fourier Transform and Series

The Fourier Transform is fundamental scientific tool that facilitates transformation of a time-dependent signal, represented by the function f(t), into a frequency-dependent function F(w) using the integral formula [8]:

$$F(w) = \int_{-\infty}^{\infty} f(t) e^{-iwt}\, dt \tag{1}$$

Where *F (w)* represents Fourier transform of time-dependent function *f (t)*, and e^{-iwt} is the Complex exponential component. This complex exponential can also be expressed as [9]:

$$e^{-iwt} = \cos(wt) - i sin(wt) \tag{2}$$

Some important mathematical identities related to Fourier analysis include [10]:

1. $\int_0^{2\pi} sin(\alpha t)\, sin(\beta t)\, dt = \begin{cases} 0 & : \alpha \neq \beta \\ \pi & : \alpha = \beta \end{cases}$
2. $\int_0^{2\pi} cos(\alpha t)\, cos(\beta t)\, dt =$ (3) $\begin{cases} 0 & : \alpha \neq \beta \\ \pi & : \alpha = \beta \end{cases}$
3. $\int_0^{2\pi} cos(\alpha t)\, sin(\beta t)\, dt = 0$

The general formula for expressing a function *f (t)* as a Fourier series is [11]:

$$f(t) = \left(\frac{\boldsymbol{a_0}}{2} + \sum_n^{\infty} \boldsymbol{a_n} cos(\beta t) + \sum_n^{\infty} \boldsymbol{b_n} sin(\beta t)\right) \tag{4}$$

Here a_0, a_n and b_n are constants that determine the coefficients of the Fourier series components.

To find these constants, we use integration as:

For b_n, *we have*

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\, sin(\alpha t) dt \tag{5}$$

Similarly For a_n, *we have*

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\, cos(\beta t) dt \tag{6}$$

And for a_0, *we have*

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(t)\, dt \tag{7}$$

Evaluating these integrals, we can determine coefficients a_0, a_n, *and* b_n and afterward use them to express any given function *f(t)* in terms of its Fourier series representation. This representation provides valuable insights into the frequency components and periodicity of the original function.

In summary, the mathematical principles of Fourier Transform and Series provide powerful tools for understanding and analyzing the frequency characteristics of time-dependent signals, enabling the representation of complex functions in terms of simpler sinusoidal components.

3. Problems

Inherent to signal transmission across various mediums is the susceptibility of signals to corruption by environmental noise. In practical scenarios, signals inevitably acquire noise during their propagation, leading to the potential degradation of the information

they carry.

To ensure the fidelity of the stored information and mitigate the adverse effects of noise, it becomes imperative to implement efficient noise reduction techniques.

4. Proposed Solution

The proposed approach centers on leveraging the Fast Fourier Transform (FFT) Method for robust noise reduction within audio signals. The method unfolds as follows:

Data Acquisition: Input audio signals are initially acquired and processed for subsequent analysis.

Noise Introduction: To simulate real-world scenarios, Additive White Gaussian Noise (AWGN) is strategically introduced to the input signals. This step aims to replicate the environmental noise that often accompanies signal transmission.

De-noising using Fourier Transform: The heart of the proposed solution lies in the application of the Fourier Transform technique to noisy signals. By utilizing this method, the inherent noise patterns are identified and subsequently filtered out, resulting in the restoration of the original signal's integrity.

Outcome Visualization: In the final phase of the proposed scheme, the efficacy of the de-noising process is visualized through the creation of waveform representations for both the original and de-noised signals. MATLAB plotting technique is applied, to construct these visualizations.

5. Methodology

Our method aims to efficiently throw out noise from audio signals while conserving their vital features. Here, we introduce the procedure employed to generate, analyze, and de-noise audio signals using MATLAB. The following are main steps involved in this method, represented by Fig. 3.

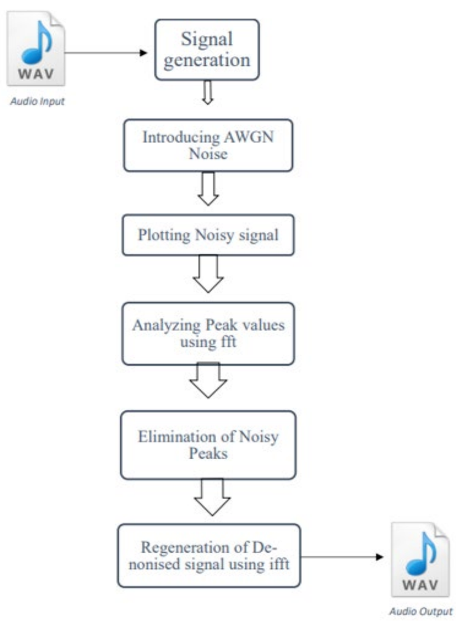


Fig. 3. Block diagram of proposed audio de-noising system

We reflected audio signal through a systematic way for better results and enhanced quality. The technique basically targets directly the peak values and process them out to clean the audio and generate results that are more efficient. In the next paras all the above-mentioned phases are further explained in detail including MATLAB coding in each step as per necessity.

A. Signal Generation and Noising

Initialy we create an audio signal which works as our foundation for analysis and de-noising. Here we took an audio file (*Impact_Moderato from freemusicarchive. org),* and convert it to a signal using MATLAB in Fig. 4. The signal contains two sinusoidal components with frequencies of 50 and 120 Hz each. By taking frequency sample of 1000 and length to 1500 for an example:

```
clc
clear all
close all
F's = 1000;   % frequency sample
Time = 1/F's;       % sampling_period
length = 1500;      % signal length
t = (0:length-1)*Time; %vector(time)
% Generate the clean audio signal
S = 0.7*sin(2*pi*50*t) + sin(2*pi*120*t);
plot(S)
title('Original Signal')
xlabel('t')
ylabel('S(t)')
```

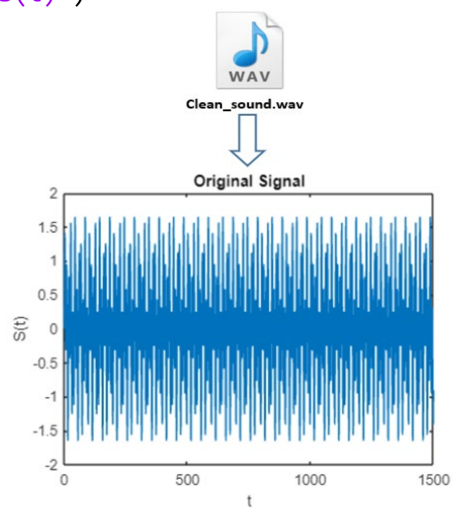


Fig. 4. Generating signal

The addition of random white noise simulates real-world noisy environments as shown in the Fig. 5, making our approach relevant to practical scenarios.

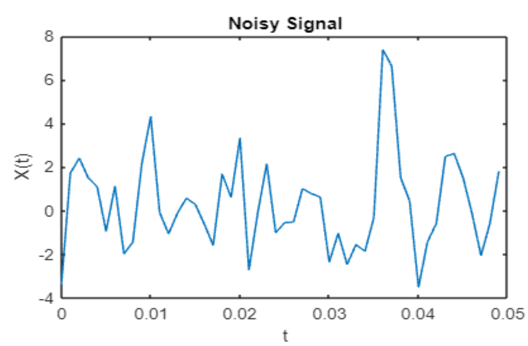


Fig. 5. Introducing white noise in signal

```
F's = 1000;         % frequency sample
Time = 1/F's;       % sampling_period
length = 1500;        % signal length
t = (0:length-1)*Time; %vector(time)
% presenting the clean audio
S = 0.7*sin(2*pi*50*t) + sin(2*pi*120*t);
% Introduce random white noise
X = S + 2*randn(size(t));
%Ploting the signal containing noise
plot(t(1:50), X(1:50))
```

```
title('Noisy Signal')
xlabel('t')
ylabel('X(t)')
```

B. Fast Fourier Transform and Spectrum Analysis

To analyze time-dependent signals in the frequency domain we use a technique of the Fast Fourier Transfom (fft) which is purely based on Mathematical Foureier Transformation. By transforming our noisy signal using fft, we can examine its frequency components. We focus on the positive amplitudes to obtain the single-sided amplitude spectrum as shown in Fig. 6.

```
Y' = fft(X);
P2 = abs(Y'/length);
P1=P2(1:length/2+1);
P1(2:end-1) = 2*P1(2:end-1);
fr = Fs*(0:(length/2))/length;
plot(fr, P1)
title('Single-Sided Amp.Spec. of X(t)')
xlabel('f (Hz)')
ylabel('|P1|')
```

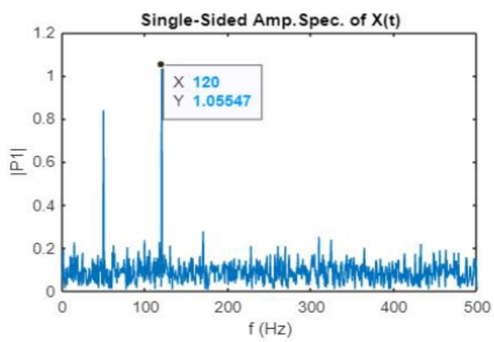


Fig. 6. Spectrum analysis of the noisy signal

C. Denoising Process

Our denoising approach involves identifying noisy frequency components and eliminating them. By observing the peaks in the single-sided spectrum, we can pinpoint the high-pitch frequency generated by the noise. Once identified, we can proceed to eliminate it to achieve denoising. Regenerated signal is represented by Fig. 7.

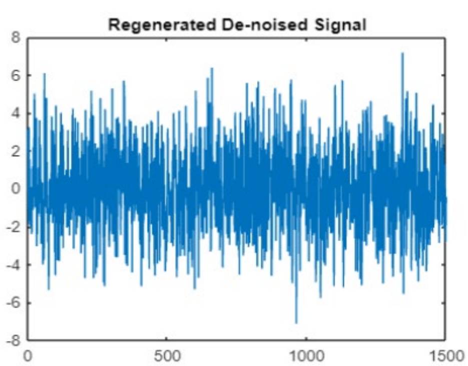


Fig. 7. Regeneration of signal after de-noising

```
% Denoising process
noisy_peak_frequency = 1;
% Id. Noise Freq.
denoised_Y' = Y';
denoised_Y'(noisy_peak_frequency)=0;
% Regenerated denoised signal
Z=ifft(denoised_Y');
subplot(2,2,3)
plot(Z)
title('Regenerated De-noised Signal')
```

D. Comparison with Original Clean Signal

For validation purposes, we compare the denoised signal with original clean signal in frequency domain by Fig. 8. This comparison provides insights into efficacy of our denoising process.

```
% Calculate the spectrum of
the original clean signal
Y'_clean = fft(S);
P2_clean = abs(Y'_clean/length);
P1_clean=P2_clean(1:length/2+1);
P1_clean(2:end-1)=2*P1_clean(2:end-1);
subplot(2,2,4)
plot(fr, P1_clean)
title('Single-Sided Amplitude
Spectrum of Clean Signal')
xlabel('f (Hz)')
ylabel('|P1|')
```

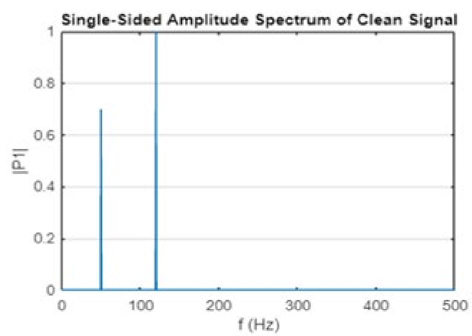


Fig. 8. Spectrum analysis of de-noised signal

6. Results and Discussion

The results of our method are highly promising, showcasing the effectiveness of our approach in eliminating noise from audio signals. The key findings from our analysis and denoising process are discussed below.

A. Analysis of Frequency Components

Upon generating the noisy signal by introducing random white noise to the clean audio signal, we employed the Fast Fourier Transform (FFT) to analyze its frequency components. The single-sided amplitude spectrum revealed the presence of multiple frequency components, including those originating from the original clean signal as well as the added noise. The comparison of the spectrum with the clean signal's spectrum underscores the impact of noise on the signal's frequency content.

B. Denoising Effectiveness

Our denoising process centered around identifying and eliminating the noisy frequency components. By pinpointing the high-pitch frequency generated by the noise, we selectively nullified the corresponding component in the frequency domain.

The regenerated denoised signal (Z) illustrates the success of this denoising technique in restoring the essential characteristics of the original audio signal, while significantly reducing the noise-related distortions.

C. Comparison with Clean Signal

For validation purposes, we compared the denoised signal's single-sided amplitude spectrum with that of the original clean signal. This comparison provided quantitative insights into the extent of noise reduction achieved by our denoising approach.

The efficacious removal of noise-related frequency components is advocated by the resemblance between the two spectra [12], featuring the robustness of our technique.

7. Conclusion

In conclusion, our method has shown an effective way to denoise signals using MATLAB. By utilizing the power of the Fast Fourier Transform and specific denoising techniques we have successfully removed noise, from signals while maintaining their characteristics. This approach has potential for improving audio processing providing signal quality and accuracy even in noisy environments.

Moving forward future research could focus on exploring optimization strategies for denoising algorithms examining the balance between reducing noise and distorting the signal and comparing our approach with techniques. The outcomes of our research open up possibilities, for audio processing methods that meet the demands of real-world situations.

List of Abbreviations

FFT: Fast Fourier Transform
AWGN: Additive White Gaussian Noise
IFFT: Inverse Fast Fourier Transform

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