International Journal of Research in Engineering, Science and Management
Volume-3, Issue-8, August-2020
journals.resaim.com/ijresm | ISSN (Online): 2581-5792 | RESAIM Publishing

Sliding Mode Controller Design for Three Tank System

K. Shrvan Venkatesh*
Department of Instrumentation and Control Engineering, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal, India
*Corresponding author: shrvanvenkateshk@gmail.com

***Abstract*: Chattering effect of sliding mode controller (SMC) is the most fundamental issue that has drawn the attention of many researchers in the area of advance control design. This paper is an attempt to address the effect of chattering in industrial process. Interacting three tank process is one of the challenging systems in industries that had requires much focus on the control design both in servo as well as regulatory operations. Level control in tanks is essential in petrochemical, paper, pharmaceutics and other industries therefore, interacting level control process is considered as working example in this paper. The performance of the system has been analyzed and results depict the efficiency of the controller.**

***Keywords*: Sliding mode control, SMC, Three tank level process, Matlab and Simulink.**

1. Introduction

Sliding mode controller (SMC) is designed for a hydraulic system that is used in pharmaceutical or chemical industries is considered. In these processes, chemical reactions are supposed to occur around pre-defined operating point therefore liquid level con- trolling is a crucial process in these industries. Sliding Mode Control (SMC) is a technique which is derived from Variable Structure Control (VSC). This control technique was originally studied by [1]. A twin tank system has been studied by several authors. In [2]-[4] the liquid is supplied to the first tank and is evacuated from the second authors proposed a method for the mathematical representation of the coupled tank system and then SMC is designed for the process. In [5] authors designed the SMC and then close loop performance of the system is compared with PID controller performance. In [6] authors proposed three sliding mode control schemes for the coupled tank system and then performances of the controlled system are studied under variations in system parameters and in the presence of external disturbance. In [7] approach for reducing chattering and testing robustness is studied for three tank system.

[8] Authors implemented sliding mode control technique for a non-linear three tanks system. The major contribution in this paper is to develop a state model and the transfer function of the plant and a sliding mode controller for liquid level control. The paper is arranged section 3 describes the three tank system and contains one subsection for mathematical modeling of three tank system. Section 4 describes SMC design for three tank system this is followed by section 5, section 6 for observation and results, conclusion.

2. System Description

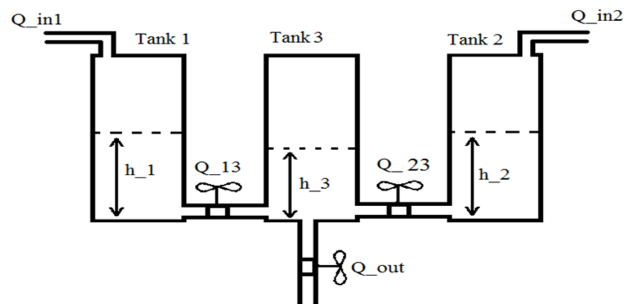

Fig. 1. Three tank system

The system considered is completely interacting three coupled tanks as shown in Fig. 1, the tanks 1 and 2 are supplied by Q_{in1} and Q_{in2} and the third is supplied from tanks 1 and 2 by two connecting pipes of cross sectional areas S_p. Through a third pipe of cross sectional area S_p, the liquid flows out. Manual valves are available between tanks 1 and 2 (Q_{12}) and the tank 3 (Q_{23}) and at the output of tank 3(Q_{out}). The liquid level in every tank is influenced by the input and output flows. As system is completely interacting liquid level in the other tanks is also influenced by liquid level in other tank. The main objective is to reach the desired level in tank 3 by controlling the input rate in tanks 1 and 2. Parameters are given in table 1.

Table 1
Parameters values of the three tank system

Symbol	Parameters	Value
Q_{in1}	Input to Tank 1	50 ml/sec
Q_{in2}	Input to Tank 3	50 ml/sec
A	Area of tank	$0.0154m^2$
S_p	Cross section of connecting pipes	$5 * 10^{-5}m$
C_1	Out flow co-efficient	1
C_2	Out flow co-efficient	1
C_3	Out flow co-efficient	0.8
g	Acceleration due to gravity	9.81 m/sec^2
h_1	liquid level in tank 1	0.22 m
h_2	liquid level in tank 2	0.22 m
h_3	liquid level in tank 3	0.20 m

3. Mathematical Modelling of Three Tank System

Assuming that $h_1 = h_2$, $h_1$and $h_2 > h_3$. The three-tank system is represented using the mass balance as given in Equation (1), (2), (3).

$$A\left(\frac{dh_1}{dt}\right) = Q_{in1} - Q_{13} \quad (1)$$
$$A\left(\frac{dh_3}{dt}\right) = Q_{13} + Q_{23} - Q_{out} \quad (2)$$
$$A\left(\frac{dh_2}{dt}\right) = Q_{in2} - Q_{23} \quad (3)$$

Where,

$Q_{13} = S_p C_1(\sqrt{2g(h_1 - h_3)}$

$Q_{23} = S_p C_2(\sqrt{2g(h_2 - h_3)}$

$Q_{out} = S_p C_3(\sqrt{2g(h_3)}$

$$\frac{dh_1}{dt} = \frac{Q_{in1} - S_p C_1(\sqrt{2g(h_1-h_3)}}{A} \quad (4)$$
$$\frac{dh_3}{dt} = \frac{S_p C_1(\sqrt{2g(h_1-h_3)} - S_p C_2(\sqrt{2g(h_2-h_3)} - S_p C_3(\sqrt{2g(h_3)}}{A} \quad (5)$$
$$\frac{dh_3}{dt} = \frac{Q_{in2} - S_p C_2(\sqrt{2g(h_2-h_3)}}{A} \quad (6)$$

The three-tank system equations involve square-root nonlinearities and the flow- rates become proportional to the square root of the tank level. In control engineering, a normal operation of the system may be around an equilibrium point and the signals may be considered as small signals around the equilibrium. However, if the system operates around an equilibrium point and if the signals involved are small signals, then it is possible to approximate the non-linear system by a linear system. Such a linear system is equivalent to the non-linear system considered within a limited operating range (Ogata 2004).

A. Line Representation

A linear model can be established around an equilibrium point using Jacobian linearisation. The linearised system is described by continues LTI representation. Where y and u represent variations around an operating point defined by the pair (U_0, Y_0). The purpose is to control the system around the operating point (U_0, Y_0), which is fixed to.

$U_o = [50 \quad 50]^T$ ml/ sec

$Y_o = [0.22 \quad 0.20 \quad 0.22]^T$ m

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (7)$$
$$y(t) = Cx(t) + Du(t) \quad (8)$$

In order to generate matrices A and B.

$$A=\begin{bmatrix} \frac{\partial f1}{\partial h1} & \frac{\partial f1}{\partial h2} & \frac{\partial f1}{\partial h3} \\ \frac{\partial f2}{\partial h1} & \frac{\partial f2}{\partial h2} & \frac{\partial f3}{\partial h3} \\ \frac{\partial f3}{\partial h1} & \frac{\partial f3}{\partial h2} & \frac{\partial f3}{\partial h3} \end{bmatrix} \quad B=\begin{bmatrix} \frac{\partial f1}{\partial Q_{in1}} & \frac{\partial f1}{\partial Q_{in2}} \\ \frac{\partial f2}{\partial Q_{in1}} & \frac{\partial f2}{\partial Q_{in2}} \\ \frac{\partial f3}{\partial Q_{in1}} & \frac{\partial f3}{\partial Q_{in2}} \end{bmatrix}$$

$$f1 = \frac{dh1}{dt}, f2 = \frac{dh2}{dt}, f3 = \frac{dh3}{dt}$$

Obtained continues time state space model is represented below.

$$A=\begin{bmatrix} -0.07981 & 0 & 0.07981 \\ 0 & -0.07981 & 0.07981 \\ 0.07981 & 0.07981 & -0.2394 \end{bmatrix}$$

$$B=\begin{bmatrix} 64.94 & 0 \\ 0 & 64.94 \\ 0 & 0 \end{bmatrix}$$

$$C = [0 \quad 0 \quad 1]$$

$$D=[0 \quad 0]$$

The transfer function matrix of the plant may be derived in terms of the state space model using the formula:

$$\bar{T}=\begin{bmatrix} T_1(s) \\ T_2(s) \end{bmatrix}=\begin{bmatrix} \frac{T_3(s)}{V_1(s)} \\ \frac{T_3(s)}{V_2(s)} \end{bmatrix} = C(sI_3 - A)^{-1} + D$$

Where $T_3(s)$, $V_1(s)$ and$V_2(s)$ are the Laplace transforms of the height $h_3(t)$ and the voltages $v_1(t)$ and $v_2(t)$.

$$\frac{T_3(s)}{V_1(s)} = \frac{5.1823s + 0.4136}{s^3 + 0.3990s^2 + 0.0318s + 0.0005}$$

$$\frac{T_3(s)}{V_2(s)} = \frac{5.1823s + 0.4136}{s^3 + 0.3990s^2 + 0.0318s + 0.0005}$$

As equation (4) and (6) are identical Q_{in1}=Q_{in2}, $C_1 = C_2$ and $h_1 = h_2$ now it can be stated that equation (4 and 6) are identical and can be rewritten in the form of equation (9).

$$\frac{dh}{dt} = \frac{Q_{in} - S_p C(\sqrt{2g(h-h_3)}}{A} \quad (9)$$
$$\frac{dh_3}{dt} = \frac{Q_{in2} - S_p C_2(\sqrt{2g(h_2-h_3)}}{A} \quad (10)$$

After linearization around an operating point.

$$A=\begin{bmatrix} -0.07981 & 0.07981 \\ 0.07981 & -0.1596 \end{bmatrix}$$

$$B=\begin{bmatrix} 64.94 \\ 0 \end{bmatrix}, C = [0 \quad 1], D=[0 \quad 0]$$

International Journal of Research in Engineering, Science and Management
Volume-3, Issue-8, August-2020
journals.resaim.com/ijresm | ISSN (Online): 2581-5792 | RESAIM Publishing

Now obtained transfer function is T(s),

$$= \frac{5.1823s + 0.4136}{s^3 + 0.3990s^2 + 0.0318s + 0.0005}$$

4. Sliding Mode Controller Design Theory

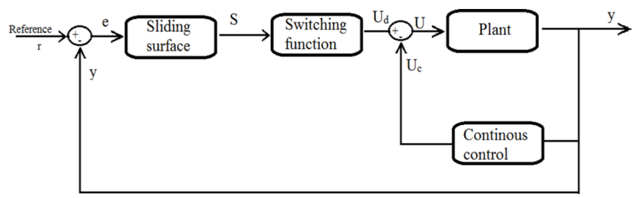

Fig. 2. Closed loop block diagram for SMC three tanks system

Above fig. 2, represents Closed loop block diagram for SMC three tanks system. Modeling inaccuracy can have strong effect on non-linear system. Particular design must address them explicitly. Sliding mode controller design provides a systematic approach to the problem of maintaining stability and consistence performance. As shown in fig. 3, idea of SMC is to chase a sliding surface along which the system can slide to its desired final value. In SMC a sliding surface has been selected at first then a suitable control law is designed so that the control variable is being driven to its references value. SMC U (t) is based on two main parts.

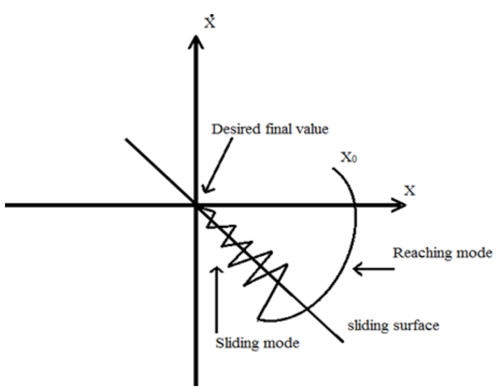

Fig. 3. State trajectory and sliding surface in SMC

1. A continuous part
2. Discontinuous part

that is $U(t) = U_c(t) + U_d(t)$

$U_c(t) = U_{eq}(t)$ is the dominated equivalent controller represents the continuous part of the controller that maintains the output the output of the system restricted to the sliding surface.

The $U_d(t)$ of SMC comprises a non-linear element that contains the switching element of the control law U_d of the controller is discontinuous across the sliding surface.

A. Sliding Mode Controller Design for Three Tank System

In SMC objective is to make the error and derivative of error equal to zero. Error is defined as the difference between actual height and desired height mathematically $e(t) = h_d(t) - h_3(t)$ where h_d is the desired liquid level and h_3 is the liquid level in tank 3 as the objective is to maintain desired liquid level in tank 3.

B. Sliding function

Most important step of SMC design is the construction of the sliding function S(t). For n^{th} order system sliding function is written as follows.

S(t) be the time varying surface then by scalar function S(x;t)=0, where

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^{(n-1)} e \quad (11)$$

In equation 11, n represents the order of the system as the plant transfer function is second order that is n=2 and sliding function for second order system can be represented as given in equation 12.

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^{1} e \quad (12)$$
$$= \dot{e} + \lambda e$$

Where λ=0 is the slope of sliding surface.

C. Stability condition

Consider lyapnov function V= $\frac{1}{2}s^2$ where $\dot{V}$ is negative definite, the system trajectory will be driven and attracted towords the sliding surface and remains sliding on it until the origin is reached asymptotically. Now $\dot{V}$=S$\dot{S}$ is the sufficient condition for stability of the system.

$$V = \frac{1}{2}\frac{d}{dt}s^2 <= -|S| \quad (13)$$

After substituting equation (12) in (13),

$$\frac{1}{2}\frac{d}{dt}(\dot{e} + \lambda e)^2 <= -|\dot{e} + \lambda e|$$

The basic discontinues control law of SMC is given by,

$$U_d = Ksgn(S)$$

Where K is constant manual tuning parameters and responsible for reaching mode the disadvantage of sliding mode controller is chattering effect, to avoid chattering effect U_d is designed as.

$$U_d = K\frac{S}{|S|+\delta} \quad (14)$$

Where chattering is solved by U_d and δ is chattering suppuration factor and is adjusted to eliminate chattering. When system remains on sliding surface that means e(t) is zero all times.

D. Continues control law $U_c(t)$

Consider equation (9 and 10)

$$\frac{dh}{dt} = \frac{Q_{in} - Q_{13}}{A} \quad (15)$$

$$\frac{dh_3}{dt} = \frac{Q_{23} + Q_{13} - Q_{out}}{A} \quad (16)$$

Where

$Q_{13} = SpC\left(\sqrt{2g(h - h3)}\right)$ for $h > h_3$

$Q_{out} = SpC_2\left(\sqrt{2g(h3)}\right)$ for $h_3 > 0$

Where h and h_3 are the liquid level of Tank 1, 2 where ($h_1 = h_2$) and Tank 3, respectively, Q_in is the inlet flow rate, A is the cross-section area of Tank. $Q_in > 0$ means that pump can only force water into the tank. Let,

$$Z_1 = h_3 > 0, Z_2 = h - h_3 > 0$$

$$C_3 = SpC_2\left(\sqrt{2g}\right), \; C = SpC\left(\sqrt{2g}\right)$$

The dynamic model in equation (15 and 16) can be written as

$$\dot{Z}_1 = -C\sqrt{Z_1} + C_3\sqrt{Z_2} \quad (17)$$

$$\dot{Z}_1 = -C\sqrt{Z_1} + C_3\sqrt{Z_2} + \frac{Q_{in}}{A} \quad (18)$$

$$Y_1 = Z_1 \quad (19)$$

Then the goal is to regulate the system output ($h_3(t)$) to the desired value (h_d) Now sliding function S can be defined as follows.

From equation (12) that is $S(t) = e^{\cdot} + \lambda e$ where e is the error that is difference between desired value(h_d) and present value(h_3). continues control law (U_c) as follows.

$$S(t) = \dot{Z}_1 + \lambda(Z_1 - h_d) \quad (20)$$

By taking the time derivative of both sides of (20),

$$\dot{S(t)} = \dot{Z}_1 + \lambda(\dot{Z}_1) \quad (21)$$

Now by using equation (17) in (21).

$$\dot{S(t)} = -C\sqrt{\dot{Z}_1} + C_3\sqrt{\dot{Z}_2} + \lambda(\dot{Z}_1) \quad (22)$$

After substituting equation (17, 18) in equation (22)

$$\dot{S} = \frac{C^2}{2} - C_3^2 - \frac{CC_3\sqrt{Z_2}}{\sqrt{Z_1}} + \frac{C_3}{\sqrt{Z_2}}\frac{Q_{in}}{A} + \lambda[2C\sqrt{Z_2} - C\sqrt{Z_1}] \quad (23)$$

$$\frac{-C_3}{\sqrt{Z_2}}\frac{Q_{in}}{A} = \frac{C^2}{2} - C_3^2 - \frac{CC_3\sqrt{Z_2}}{\sqrt{Z_1}} + \lambda[2C\sqrt{Z_2} - C\sqrt{Z_1}] - \dot{S} \quad (24)$$

Where,

$$\dot{S} = -Ksig(s)$$

$$Q_{in} = \frac{A}{C_3}\sqrt{Z_2}\left[\frac{C^2}{2} - C_3^2 - \frac{CC_3\sqrt{Z_2}}{\sqrt{Z_1}} + \lambda[2C\sqrt{Z_2} - C\sqrt{Z_1}]\right] + Ksig(s) \quad (25)$$

Above equation (25) gives the continuous control law $U_c(t)$ Control variable $U(t) = U_c(t) + U_d(t)$

$$Q_{in} = \frac{A}{C_3}\sqrt{Z_2}\left[\frac{C^2}{2} - C_3^2 - \frac{CC_3\sqrt{Z_2}}{\sqrt{Z_1}} + \lambda[2C\sqrt{Z_2} - C\sqrt{Z_1}]\right] + \frac{S}{|S| + \delta} \quad (26)$$

5. Results Analysis

Three tank level processes is a challenging interacting process. The effect of tank-1 influences the change in level of tank-2 and tank-3. Similarly, the effect of tank-3 influences the variation in tank-1 and tank-2. In this process, tank-2 is an interacting level.

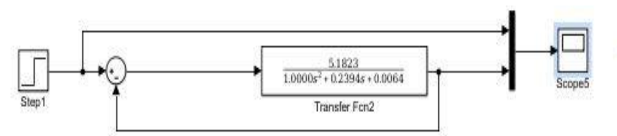

Fig. 4. Closed loop Simulink model

Above fig. 4, represent the close loop simulink model of three tank system. Fig. 5, represents the response of conventional controller close loop reference tracking response and it is observed that response have oscillations in the initial phase of response. It shows that, there is requirement of advanced controller to reduce the oscillations. Hence, to suppress these oscillations, SMC has been implemented on the system.

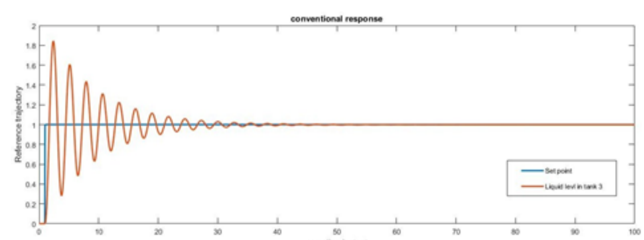

Fig. 5. Closed loop response

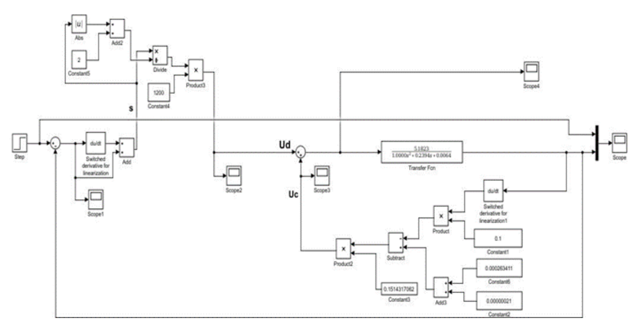

Fig. 6. Sliding mode controller simulink model

Fig. 6, shows the sliding mode controller simulink model for three tank system plant. The simulation results for controller with K=1200, δ = 0.1 and λ = 2 has shown in Fig. 7. It is observed that performance of SMC shown efficient compared to conventional controller.

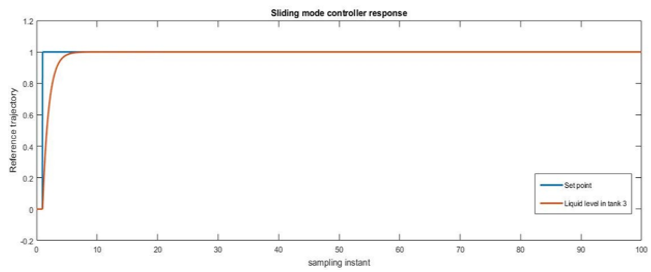

Fig. 7. Sliding mode controller response

6. Conclusion

Mathematical modeling of three tank level process has been carried out using physics based modeling. The sliding mode control technique was implemented for a non- linear three tanks system. New state model and the transfer function of the plant was developed and chattering effect obtained in conventional controller is reduced by implementing SMC to three tank liquid level control process. It is observed that SMC is able to control the system with its robust control behavior for different input signal and performance of controller is observed to be minimum in manipulated input variable.

References

[1] V. Utkin, "Variable structure systems with sliding modes," IEEE Transactions on Automatic control, vol. 22, no. 2, pp. 212–222, 1977.
[2] K. Narwekar and V. Shah, "Level control of coupled tank using higher order sliding mode control," in *2017 IEEE International Conference on Intelligent Techniques in Control, Optimization and Signal Processing (INCOS)*, IEEE, 2017, pp. 1–5.
[3] H. Mamur, I. Atacak, F. Korkmaz, and M. Bhuiyan, "Modelling and application of a computer-controlled liquid level tank system," in *Proc. Sixth International Conference on Embedded Systems and Applications (EMSA 2017)*, 2017, pp. 97– 106.
[4] H. Delavari and A. Ranjbar, "Robust intelligent control of coupled tanks," in *Proceedings of the 9th WSEAS International Conference on Automatic Control, Modeling and Simulation*, Istanbul, Turkey, 2007, pp. 1–6.
[5] B. Parvat, V. Jadhav, and N. Lokhande, "Design and implementation of sliding mode controller for level control," *IOSR Journal of Electronics and Communication Engineering (IOSR-JECE)*, pp. 51–54, 2012.
[6] N. B. Almutairi and M. Zribi, "Sliding mode control of coupled tanks," *Mechatronics*, vol. 16, no. 7, pp. 427–441, 2006.
[7] R. Benayache, L. Chrifi-Alaoui, P. Bussy, and J. Castelain, "Nonlinear sliding mode control with backstepping approach for a nonlinear three tank system," in *2008 16th Mediterranean Conference on Control and Automation. IEEE*, 2008, pp. 658–663.
[8] D. Chirit¸ Aˇ, A. Florescu, B. C. Florea, R. ENE, and D. A. Stoichescu, "Liquid level control for industrial three tanks system based on sliding mode control," 2015.