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Fuzzy Simple, Fuzzy Identity and Fuzzy Zero of a Po-Ternary Gamma Semi Group

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***Abstract*: We introduce the terms Fuzzy Simple POTΓSG, Fuzzy left (right & lateral) identity. And proved Fuzzy left (right& lateral) identity of a POTΓSG if exists they are same. Fuzzy left (right & lateral) zero of a POTΓSG Also shows that the intersection of infinite family of fuzzy POTΓSSGs of a POTΓSG T is fuzzy POTΓSG of T. Also *h* is a fuzzy ideal of a POTΓSG T iff $h\Gamma T\Gamma T \subseteq h$, $T\Gamma T\Gamma h \subseteq h$, $T\Gamma h\Gamma T \subseteq h$ and $(h] \subseteq h$. Finally, we showed the intersection, union of arbitrary family of fuzzy deals of T is an ideal of T.**

***Keywords*: Fuzzy simple POTΓSG, Fuzzy identity of a POTΓSG, Fuzzy zero of a POTΓSG, Fuzzy ideal.**

1. Introduction

A. H. Clifford and Preston G.B [2], [3], Petrich. M [5] and Ljapin E. S [4] were deeply studied Algebraic theory of semi groups. A.Anjaneyulu [1] had buildout an ideal theory in semi groups. The "fuzzy theory" of semi groups are established by Kuroki and Xie. Sarala.Y [13] defined theory of ideals in ternary semigroup. Pradeep J.M, Gangadhararao. A, Ramyalatha. P, Achala [16] defined Fuzzy identity and Fuzzy zero of a PO ternary semi group, PO ideal, PO ideal generated by a subset.

We introduce some classical concepts of Fuzzy simple POTΓSG, Fuzzy identity and Fuzzy zero of a POTΓSG, operations on fuzzy POTΓSG in this paper. We denote Po ternary Γ semi group as POTΓSG, Po ternary Γ sub semi group as POTΓSSG, Fuzzy Subset as FS, Fuzzy ideal as FI, ordered fuzzy point as OFP, fuzzy left ideal as FLI, fuzzy right ideal as FRI throughout in this paper.

2. Prerequisites

Definition 2.1: "A semi group T has an ordered relation "$\leq$" is known as PO ternary Γ semi group (POTΓSG) if T is a POSET such that $q \leq r \Rightarrow q\gamma q_1\delta q_2 \leq r\gamma q_1\delta q_2$, $q_1\gamma q\delta q_2 \leq q_1\gamma r\delta q_2$, $q_1\gamma q_2\delta q \leq q_1\gamma q_2\delta r$ $\forall$ $q, r, q_1, q_2 \in T$".

Definition 2.2: "Let $\phi \neq C \subseteq T$. The characteristic mapping $h_C : T \to [0,1]$ is defined as $h_C(t) = \begin{cases} 1 & \text{if } t \in C \\ 0 & \text{if } t \notin C \end{cases}$. Then h_C, is a fuzzy subset (FS) of T".

Definition 2.3: "A mapping $h : T \to [0,1]$ is said to be a FS of T. The POTΓSG itself a FS of $T \ni T(t) = 1\ \forall t \in T$ and it denotes T or 1".

Definition 2.4: Let K $\subseteq$T**,** a POTΓSG. Now let us define $(K] = \{r \in T / \ r \leq h \text{ for some } h \in K\}$, when $K = \{c\}$ we write $(c] = (\{c\}] = \{r \in T / r \leq c\}$.

Definition 2.5: Let K $\subseteq$T**,** a POTΓSG. Now let us define $[K) = \{r \in T / \ h \leq r \text{ for some } h \in K\}$, when $K = \{c\}$ we write $(c] = (\{c\}] = \{r \in T / r \leq c\}$.

Definition 2.6: Let *h* be a FS of a POTΓSG T. Let us define (*h*] as $(h](r) = \vee_{r \leq s} h(s)$, $\forall$ r$\in$T

Note 2.7: "The collection of every FSs of T is denoted as H(T)".

Definition 2.8: Let u, v, w be FS of a POTΓSG T. For each $t \in T$ the product $u\Gamma v\Gamma w$ is defined as
$$(u\Gamma v\Gamma w)(t) = \begin{cases} \vee_{t \leq q\gamma r\delta s} u(q) \wedge v(r) \wedge w(s) & \text{if } t \leq q\gamma r\delta s \text{ exists} \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.10: "Let h be FS of T, a POTΓSG is known as fuzzy POTΓSSG of T if $(i)\ q \leq r$ then $h(q) \geq h(r)$ (ii) $h(q\gamma r\delta s) \geq h(q) \wedge h(r) \wedge h(s),\ \forall q, r, s \in T, \gamma, \delta \in \Gamma$".

Definition 2.11: "a FS h of a POTΓSG T is known as fuzzy PO left ideal of T if
$(i)\ q \leq r$ then $h(q) \geq h(r)$
(ii) $h(q\gamma r\delta s) \geq h(s),\ \forall q, r, s \in T, \gamma, \delta \in \Gamma$".

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Lemma 2.12: "Let T be a POTΓSG and h be FS of T. h is a fuzzy PO left ideal of $\mathrm{T} \Leftrightarrow h$ satisfies the conditions that $(i)\, q \le r$ then $h(q) \ge h(r),\ \forall q, r \in \mathrm{T}$,
$(ii)\ \mathrm{T}\Gamma h \Gamma h \subseteq h$".

Definition 2.13: "a FS h of a POTΓSG T is known as fuzzy PO right ideal of T if
$(i)\, q \le r$ then $h(q) \ge h(r)$
$(ii)\ h(q\gamma r\delta s) \ge h(q),\ \forall q, r, s \in \mathrm{T}, \gamma, \delta \in \Gamma$".

Lemma 2.14: Let h be a FS of a POTΓSG T. h is a fuzzy PO right ideal of T iff h satisfies the conditions that
$(i)\, q \le r$ then $h(q) \ge h(r),\ \forall q, r \in \mathrm{T}$, $(ii)\ h\Gamma h \Gamma \mathrm{T} \subseteq h$.

Definition 2.15: "a FS h of a POTΓSG T is known as fuzzy PO lateral ideal of T if
$(i)\, q \le r$ then $h(q) \ge h(r)$
$(ii)\ h(q\gamma r\delta s) \ge h(r),\ \forall q, r, s \in \mathrm{T}, \gamma, \delta \in \Gamma$".

Lemma 2.16: Let h be a FS of a POTΓSG T. h is a fuzzy PO lateral ideal of T iff h satisfies the conditions that $(i)\, q \le r$ then $h(q) \ge h(r),\ \forall q, r \in \mathrm{T}$, $(ii)\ h\Gamma \mathrm{T} \Gamma h \subseteq h$.

Definition 2.17: "Let T be a POTΓSG and h be a FS of T is said to be a 'fuzzy ideal(FI)' of T if
$(i)\, q \le r$ then $h(q) \ge h(r)$
$(ii)\ h(q\gamma r\delta s) \ge h(s),\ h(q\gamma r\delta s) \ge h(q),\ h(q\gamma r\delta s) \ge h(r)$.

Lemma 2.18: "Let T be a POTΓSG and h be a FS of T. Then, h is FI of T iff h satisfies the conditions that
$(i)\, q \le r$ then $h(q) \ge h(r),\ \forall q, r \in \mathrm{T}$,
$(ii)\ \mathrm{T}\Gamma h \Gamma h \subseteq h, h\Gamma h \Gamma \mathrm{T} \subseteq h, h\Gamma \mathrm{T} \Gamma h \subseteq h$".

Lemma 2.19: Let $\phi \ne \mathrm{C} \subseteq \mathrm{T}$, a POTΓSG. Then C is a left ideal of T iff the characteristic function h_{C}, of C is a FLI of T.

Lemma 2.20: Let T be a POTΓSG and $\phi \ne \mathrm{C} \subseteq \mathrm{T}$. Then C is a right ideal of T iff the characteristic function h_{C}, of C is a FRI of T.

Lemma 2.21: Let T be a POTΓSG and $\phi \ne \mathrm{C} \subseteq \mathrm{T}$. Then C is an ideal of T iff the characteristic function h_{C}, of C is a FI of T.

Proposition 2.22: Let h, g, s be three FSs of T. Then the subsequent conditions are true.

1. $h \subseteq (h], \forall h \in H(T)$
2. If $h \subseteq g$, then $(h] \subseteq (g]$
3. $(h]\Gamma(g] \subseteq (h\Gamma g),\ \forall\ h, g \in H(T)$
4. For any FI h of T, $(h] \subseteq (g]$
5. If h, g are FI of T, then $h\Gamma g$, $h \cup g$ are FIs of T
6. $h\Gamma(g \cup f] \subseteq (h\Gamma g \cup h\Gamma f]$
7. $(g \cup h]\Gamma f \subseteq (g\Gamma f \cup h\Gamma f]$
8. If z_λ is an OFP of T, then $z_\lambda = (z_\lambda]$.

Definition 2.23: Let T be a POTΓSG, $z \in T$ and $\lambda \in (0,1]$. An OFP z_λ, $z_\lambda : T \to [0,1]$ defined by

$$z_\lambda(r) = \begin{cases} \lambda & \text{if } r \in (z] \\ 0 & \text{if } r \notin (z] \end{cases}$$

Clearly z_λ is a FS of T. For each FS h of T and denote $z_\lambda \subseteq h$ as $z_\lambda \in h$

Definition 2.24: Let h be a FS of T and $t \in [0,1]$. If $h_t = \{r / r \in T / h(r) \ge t\}$ then h_t is termed as t-cut or a level set.

3. Fuzzy Simple POTΓSG

Definition 3.1: "A POTΓSG T is known as left Simple Ternary Γ semi group (LSTΓSG) if T is itself only left ideal".

Definition 3.2: "A POTΓSG T is known as fuzzy left Simple T Γ semi group (FLSTΓSG) if each fuzzy left ideal (FLI) in T is a constant function".

Definition 3.3: Let h be a FS of a POTΓSG T. We define $h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(r)$ as

$$h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(r) = \begin{cases} 1 & \text{for } r \in (\mathrm{T}\Gamma\mathrm{T}\Gamma a] \\ 0 & \text{otherwise} \end{cases}$$

Theorem 3.4: "Let T be a POTΓSG. Then $h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}$ is FLI of T, for each $a \in \mathrm{T}$".

Proof: (1) for $q, r, s \in \mathrm{T}$ and

If $r \in (\mathrm{T}\Gamma\mathrm{T}\Gamma a]$, then

$h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(q) = h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(r) = h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(s) = 1$ since $q \le r \Rightarrow q \in (\mathrm{T}\Gamma\mathrm{T}\Gamma a]$.

If $r \notin (\mathrm{T}\Gamma\mathrm{T}\Gamma a]$, then

$h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(r) = 0 \le h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(s) = 1$.

By summarizing the above $h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(q) \ge h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(r)$.

(2) If $r \notin (\mathrm{T}\Gamma\mathrm{T}\Gamma a]$, then

$h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(r) = 0 \le h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(q\gamma r\delta s)$.

If $r \in (\mathrm{T}\Gamma\mathrm{T}\Gamma a]$, then $h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(r) = 1$

$\because r \in (\mathrm{T}\Gamma\mathrm{T}\Gamma a]$ and $(\mathrm{T}\Gamma\mathrm{T}\Gamma a]$ is a PO right ideal of T, then

$q\gamma r\delta s \in (\mathrm{T}\Gamma\mathrm{T}\Gamma a] \forall q \in \mathrm{T}, \gamma, \delta \in \Gamma$

$\Rightarrow h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(q\gamma r\delta s) = 1 = h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(s)$

$h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(q\gamma r\delta s) \ge h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}(s)$

From (1) and (2), $h_{(\mathrm{T}\Gamma\mathrm{T}\Gamma a]}$ is FLI.

Theorem 3.5: Let T be a POTΓSG, the subsequent statements are equal.

1) T is a LSPOTΓSG 2) T is a FLSTΓSG.

Proof: First we prove (i) $\Rightarrow$ (ii):

Assume that T is a LSPOTΓSG.

Suppose h is any FLI of T, Then $q, r, s \in \mathrm{T}$ and

$\gamma, \delta \in \Gamma$

$m = q\gamma l\delta l$ and $l = r\gamma m\delta m$

By definition of T,

we have $h(l) = h(r\gamma m\delta m) \geq h(m) = h(q\gamma l\delta l) \geq h(l)$

$\therefore h(l) = h(m)$

$\therefore h$ is constant FI.

Hence, T is fuzzy left simple POTΓSG.

(ii)$\Rightarrow$ (i): Suppose that T is a FLSTΓSG.

If C is any PO left ideal, then C_c is a FLI of T.

$\Rightarrow C_C$ is constant function.

Let $r \in T$ and $C \neq \varphi, C_C(r) = 1$ implies that $r \in C$

$\Rightarrow T \subseteq C.$

$\therefore T = C.$ Hence T is LSPOTΓSG.

Theorem 3.6: Let T be a POTΓSG. Then T is a FLSPOTΓSG iff $h_{(T\Gamma T\Gamma a]} = T = h_T \ \forall a \in T.$

Proof: Suppose that T is a FLSPOTΓSG.

By above theorem, T is a LSPOTΓSG. Then, we have $(T\Gamma T\Gamma a] = T.$

Therefore $h_{(T\Gamma T\Gamma a]} = T = h_T \ \forall a \in T.$

Conversely, suppose that $h_{(T\Gamma T\Gamma a]} = T = h_T$.

$\Rightarrow h_{(T\Gamma T\Gamma a]}(t) = h_T(t)$

$\Rightarrow (T\Gamma T\Gamma a] = T.$ Then T is a LSPOTΓSG**.** Then by above theorem, T is a FLSPOTΓSG.

Definition 3.7: **"**A POTΓSG T is known as right Simple Ternary Γ semi group (RSTΓSG) if T is itself only PO right ideal".

Definition 3.8: **"**A POTΓSG T is known as fuzzy right Simple Ternary Γ semi group (FRSTΓSG) if each FRI of T is constant function".

Definition 3.9: Let h be a FS of a POTΓSG T. We define

$$h_{(a\Gamma T\Gamma T]}(r) = \begin{cases} 1 & \text{for} \quad r \in (a\Gamma T\Gamma T] \\ 0 & \text{otherwise} \end{cases}$$

Definition 3.10: **"**Let T be a POTΓ SG T is known as fuzzy Simple ternary Γ semi group (**FSTΓSG)** if every FI of T is a constant function".

Theorem 3.11: **"**Let T be a POTΓSG. Then $h_{(a\Gamma T\Gamma T]}$ is a FRI of T for all $a \in T$".

Proof: (a) Let $p, q, r \in T$ and $\alpha, \beta \in \Gamma$

Also $p \leq q, q \leq r.$

If $q \in (a\Gamma T\Gamma T],$ then

$h_{(a\Gamma T\Gamma T]}(p) = h_{(a\Gamma T\Gamma T]}(q) = h_{(a\Gamma T\Gamma T]}(r) = 1$ since $p \leq q$ implies $p \in (a\Gamma T\Gamma T]$

If $q \notin (a\Gamma T\Gamma T]$ then $h_{(a\Gamma T\Gamma T]}(q) = 0 \leq h_{(a\Gamma T\Gamma T]}(p)$

By summarizing the above $h_{(a\Gamma T\Gamma T]}(p) \geq h_{(a\Gamma T\Gamma T]}(q).$

(b) If $p \notin (a\Gamma T\Gamma T],$ then

$h_{(a\Gamma T\Gamma T]}(p) = 0 \leq h_{(a\Gamma T\Gamma T]}(p\alpha q\beta\gamma)$

If $p \in (a\Gamma T\Gamma T],$ then $f_{(a\Gamma T\Gamma T]} = 1.$

Since $p \in (a\Gamma T\Gamma T]$ and $(a\Gamma T\Gamma T]$ is PO right ideal, then $p \in (a\Gamma T\Gamma T] \in p\alpha q\beta\gamma$ for all $q \in T$ and $\alpha, \beta \in \Gamma$

$\therefore h_{(a\Gamma T\Gamma T]}(p\alpha q\beta r) = 1 = h_{(a\Gamma T\Gamma T]}(p)$

Therefore $h_{(a\Gamma T\Gamma T]}(p\alpha q\beta r) \geq h_{(a\Gamma T\Gamma T]}(p)$.

$\therefore h_{(a\Gamma T\Gamma T]}$ is a FRI of T .

Theorem 3.12: Let T be a POTΓSG T, then the subsequent conditions are equal.

(1) T is a RSPOTΓSG 2) T is a FRSPOTΓSG

Proof: (1)$\Rightarrow$ (2):

Suppose T is a RSPOTΓSG.

Consider h is any FRI, Then $l, m, n \in T$ and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $p\alpha p\beta l = q$ and $q\gamma q\delta m = p.$

Since h is a FRI, Then

$h(p) = h(q\gamma q\delta m) \geq h(q) = h(p\alpha p\beta m) \geq f(p)$

$\Rightarrow h(p) = h(q) \forall p, q \in T$

h is a constant FI.

Hence T is a FRSPOTΓSG.

(2)$\Rightarrow$ (1): Suppose that T is a FRSPOTΓSG.

Let A be any PO 'right ideal' of T. Then C_A is a FRI.

$\Rightarrow C_A$ is a constant function.

Let $t \in T$, since $A \neq \varphi, C_A(t) = 1$ implies that $t \in A$

$\Rightarrow T \subseteq A.$

$T = A.$

$\therefore$ T is a RSPOTΓ SG.

Theorem 3.13: "Let T be a POTΓSG. T is a FRSPOTΓSG $\Leftrightarrow$ $h_{(a\Gamma T\Gamma T]} = T = h_T \ \forall a \in T$".

Proof: Suppose that T is a FRSPOTΓSG. By above Theorem, T is a RSPOTΓSG. Then, we have $(a\Gamma T\Gamma T] = T.$

Therefore $h_{(a\Gamma T\Gamma T]} = T = h_T \ \forall a \in T.$

Conversely assume that $h_{(a\Gamma T\Gamma T]} = T = h_T$

$\Rightarrow h_{(a\Gamma T\Gamma T]}(t) = h_T(t)$

$\Rightarrow (a\Gamma T\Gamma T] = T.$ Then, T is a RSPOTΓSG. Then by above Theorem, T is a FRSPOTΓSG.

Definition 3.14: **"**Let u, v, w be three FSs of T, $(u\Gamma v\Gamma w]$ is defined by $(u\Gamma v\Gamma w](t) = \bigvee_{t \leq r\gamma s\delta w} (u\Gamma v\Gamma w)(p\gamma q\delta r),$ $\forall t \in T, \gamma, \delta \in \Gamma$".

Definition 3.15: **"**Let T be a POTΓSG and h be FI of T. Then h is known as globally idempotent if $(h^n] = (h], \forall n$".

Definition 3.15: Let T be a POTΓSG. T is known as fuzzy globally idempotent if $(T^n] = (T], \forall n.$

Theorem 3.16: **"**If T is a POTΓSG with unity "e" and h is a FI of "T" with $h(e)=1,$ then $h=h_{\mathrm{T}}=\mathrm{T}$".

Proof: Let $r\in \mathrm{T}_{\mathrm{T}}$.

Consider $h(r)=h(r\gamma e\delta e)\geq h(e)=1.\therefore h=h_{\mathrm{T}}=\mathrm{T}.$

Definition 3.17: a non-zero FI *h* of POTΓSG T is known as proper FI if $h\neq \mathrm{C}_{\mathrm{T}}=\mathrm{T}.$

Theorem 3.18: **"**Let $\{h_i\}$ be any FIs of a POTΓSG T. Then the infinite union of FIs is FI of T"**.**

Proof: let $\{h_i\}$ is a FIs of a POTΓSG T.

Let $q,r,s\in \mathrm{T}$ such that $q\leq r, r\leq s.$

Consider $\cup h_i(q)=\max\{h_1(q),h_2(q),h_3(q),...\}$

$=h_1(q)\vee h_2(q)\vee h_3(q)\vee ...$

$\geq h_1(r)\vee h_2(r)\vee h_3(r)\vee ...$ since each h_i is a FI.

$=\max\{h_1(r),h_2(r),h_3(r),...\}=\cup h_i(r)$

$\therefore \cup h_i(q)\geq \cup h_i(r)$ if $q\leq r.$

Consider

$\cup h_i(q\gamma r\delta s)=h_1(q\gamma r\delta s)\vee h_2(q\gamma r\delta s)\vee h_3(q\gamma r\delta s)\vee ...$

. $\geq h_1(r)\vee h_2(r)\vee h_3(r)\vee ...$ since each h_i is a "fuzzy lateral ideal".

$=\cup h_i(r).$

$\therefore \cup h_i(q\gamma r\delta s)\geq \cup h_i(r).$

Similarly, $\cup h_i(q\gamma r\delta s)\geq \cup h_i(q)$ and $\cup h_i(q\gamma r\delta s)\geq \cup h_i(s).$

Hence, $\cup h_i$ is a FI of T.

Definition 3.19: Let T be a POTΓSG and h be a FI of T. h is said to be a maximal if there does not have any proper FI g of T $\ni h\subset g.$

Theorem 3.20: **"**If T is a POTΓSG with unity e, then the union of all proper FIs of T is the unique fuzzy maximal ideal of T"**.**

Proof: Suppose h_{M} is the union of all proper FIs of T.

$\Rightarrow h_{\mathrm{M}}$ is a FI of T

Consider h_{M} is not proper then

$h_{\mathrm{M}}=\mathrm{C}_{\mathrm{T}}\Rightarrow h_{\mathrm{M}}(x)=1\forall x\in \mathrm{T}$

$h_i(x)=1$ for some FI h_i

$\because \cup h_i=h_{\mathrm{M}}\Rightarrow h_i=h_{\mathrm{T}}$ but h_i is proper.

Hence h_{M} is a proper FI of T.

$\because h_{\mathrm{M}}$ contains all proper FIs of T.

$\Rightarrow h_{\mathrm{M}}$ is maximal FI of T.

If g_{M} is any other maximal FI of T, then $g_{\mathrm{M}}\subseteq h_{\mathrm{M}}\subseteq \mathrm{C}_{\mathrm{T}}$.

$\therefore g_{\mathrm{M}}=h_{\mathrm{M}}$.

Hence, h_{M} is the unique 'fuzzy maximal ideal' of T.

Theorem 3.21: Let T be a fuzzy left STΓSG, then T is fuzzy simple Γ semi group.

Proof: Assume that T is a fuzzy left STΓSG.

consider h is a FI of T

$\therefore h$ is a FLI of T.

Hence h is a constant function

Therefore, T is a fuzzy simple Γ semi group.

Corollary3.22: **"**Let T be a fuzzy lateral (right) STΓSG, then T is fuzzy simple Γ semi group".

Theorem 3.23: let T be a POTΓSG and c_λ be a "Ordered Fuzzy Point"(OFP) of T. If c_λ is semi simple and idempotent, Then $c_\lambda\subseteq <c_\lambda>^n,\forall n.$

Proof: If c_λ is semi simple and idempotent.

Let $c\in \mathrm{T}$ and n is a natural number.

$\Rightarrow c_\lambda\subseteq <c_\lambda>^3$ is true for $n=3$ since c_λ is fuzzy semi simple.

Consider, the assumption is true for

$n-2.$ *i.e.*, $c_\lambda\subseteq <c_\lambda>^{n-2}$

suppose $<c_\lambda>^{n-2}\Gamma<c_\lambda>\Gamma<c_\lambda>$

$\supseteq c_\lambda\Gamma c_\lambda\Gamma c_\lambda=c_\lambda{}^3=c_\lambda,$

c_λ is idempotent. Therefore $c_\lambda\subseteq <c_\lambda>^n,\forall n.$

4. Fuzzy Identity and Fuzzy Zero of a POTΓSG

Definition 4.1: Let T be a POTΓSG and *h* be FS of T. Let us define [*h*) by $[h)(r)=\bigvee_{r\geq s} h(s),\forall r\in T$ where $s\in T.$

Proposition 4.2: Let *h*, g be FSs of T. The subsequent conditions are true.

1) $h\in T$ 2) if $h\subseteq g$ then $[h)\subseteq [g).$

Proof: 1) let $r\in T$, Since

$[h)(r)=\bigvee_{r\geq s}[h)(s),\forall s\in T$

Since $r\geq r\Rightarrow [h)(r)=\bigvee_{r\geq r}h(r)\geq h(r).$

Hence $h\subseteq [h)$

2) If $h\subseteq g$ then $\forall r\in T, h(r)\leq g(r)$,

Thus

$[h)(r)=\bigvee_{r\geq s}(h)(s)\leq \bigvee_{r\geq s} g(s)=[g)(r),\forall r\in T.$

Hence $[h)\subseteq [g)$.

DEFINITION 4.3: An OFP z_λ of a POTΓSG T is known as fuzzy left identity of T if $z_\lambda\Gamma h\Gamma h=h$ and

$h \subseteq z_\lambda, \forall h \in H(T), z \in T$ and $\lambda \in (0,1]$.

DEFINITION 4.4: An OFP z_λ of a POTΓSG T is known as fuzzy right identity of T if $h\Gamma h\Gamma z_\lambda = h$ and $h \subseteq z_\lambda, \forall h \in H(T), z \in T$ and $\lambda \in (0,1]$. .

DEFINITION 4.5: An OFP z_λ of a POTΓSG T is known as fuzzy lateral identity of T if $h\Gamma z_\lambda \Gamma h = h$ and $h \subseteq z_\lambda, \forall h \in H(T), z \in T$ and $\lambda \in (0,1]$.

DEFINITION 4.6: A FS *h* of a POTΓSG T with an identity is called as fuzzy left identity of T if $h\Gamma h_1 \Gamma h_2 = h$ and $h_1 \subseteq h, h_2 \subseteq h \forall h_1, h_1 \in H(T)$.

DEFINITION4.7: A FS *h* of a POTΓSG T with identity is called as fuzzy lateral identity of T if $h_1 \Gamma h \Gamma h_2 = h$ and $h_1 \subseteq h, h_2 \subseteq h \forall h_1, h_2 \in H(T)$.

DEFINITION4.8: A FS *h* of a POTΓSG T with identity is called as fuzzy right identity of T if $h_1 \Gamma h_2 \Gamma h = h$ and $h_1 \subseteq h, h_2 \subseteq h \forall h_1, h_2 \in H(T)$.

DEFINITION4.9: An OFP z_λ of a POTΓSG T is called as fuzzy zero of T if $z_\lambda \Gamma h \Gamma h = h \Gamma z_\lambda \Gamma h = h \Gamma h \Gamma z_\lambda = z_\lambda$ and $h \subseteq z_\lambda, \forall h \in H(T)$.

THEOREM4.10: If q_λ is a fuzzy PO left zero, r_λ is a fuzzy PO right zero and s_λ PO lateral zero of a POTΓSG T then $q_\lambda = r_\lambda = s_\lambda$ where $\lambda \in [0,1]$.

Proof: Given q_λ is fuzzy PO left zero of T .

$\therefore q_\lambda \Gamma h \Gamma g = q_\lambda \forall h, g \in H(T)$ and $q_\lambda \subseteq h$

$\therefore q_\lambda \Gamma s_\lambda \Gamma r_\lambda = q_\lambda$ and $q_\lambda \subseteq h$, $\forall h \in H(T)$.

since r_λ is a fuzzy PO right zero of T

$\Rightarrow q_\lambda \Gamma s_\lambda \Gamma r_\lambda = r_\lambda$ and $r_\lambda \subseteq g, \forall g \in H(T)$.

Since s_λ is a fuzzy PO lateral zero of T

$\therefore h \Gamma s_\lambda \Gamma g = s_\lambda \forall h, g \in H(T)$

$\Rightarrow q_\lambda \Gamma s_\lambda \Gamma r_\lambda = s_\lambda$ and $s_\lambda \subseteq h, \forall h \in H(T)$.

$\therefore q_\lambda \Gamma s_\lambda \Gamma r_\lambda = q_\lambda = r_\lambda = s_\lambda$.

THEOREM 4.11: "Let T be a fuzzy POTΓSG. Then T has atmost one fuzzy zero element".

Proof: let $q_\lambda, r_\lambda, s_\lambda$ be any 3 fuzzy zeros of a POTΓSG T.

$\Rightarrow q_\lambda, r_\lambda, s_\lambda$ be treated as fuzzy left, lateral & right zeros of T resp.

We know that by the above theorem, we have $q_\lambda = r_\lambda = s_\lambda$.

Hence a fuzzy POTΓSG has at most one fuzzy PO zero element.

5. Operations on Fuzzy POTΓSG

Definition 5.1: Let $\{h_i\}_{i \in I}$ be the family of FSs of a POTΓSG T and I,an index set. Now define intersection, union as follows.

$$(\bigcap_{i \in I} h_i)(r) = \bigwedge_{i \in I} h_i(r) = \min\{h_1(r), h_2(r), h_3(r) ----\}, \forall r \in T,$$

$$(\bigcup_{i \in I} h_i)(r) = \bigvee_{i \in I} h_i(r) = \max\{h_1(r), h_2(r), h_3(r), ---\}, \forall r \in T.$$

Definition 5.2: "a FS h of a POTΓSG T is known as fuzzy POTΓSG of T if $(i)\, q \leq r$ then $h(q) \geq h(r)$ (ii) $h(q\gamma r\delta s) \geq h(q) \wedge h(r) \wedge h(s),\ \forall q, r, s \in \mathrm{T}, \gamma, \delta \in \Gamma$".

Theorem 5.4: "The intersection of any two fuzzy POTΓSSGs of a POTΓSG T is a fuzzy POTΓSSG of T".

Proof: If h_1, h_2 be any 2 fuzzy POTΓSSG of T.

1) Suppose

$$(h_1 \cap h_2)(q\Gamma r\Gamma s) = h_1(q\Gamma r\Gamma s) \wedge h_2(q\Gamma r\Gamma s) \geq h_1(q) \wedge h_1(r) \wedge h_2(q) \wedge h_2(r) \wedge h_1(s) \wedge h_2(s)$$

$$\geq h_1(q) \wedge h_2(q) \wedge h_1(r) \wedge h_2(r) \wedge h_1(s) \wedge h_2(s)$$

$$\geq (h_1 \cap h_2)(q) \wedge (h_1 \cap h_2)(r) \wedge (h_1 \cap h_2)(s), \forall q, r, s \in T.$$

2) Let $q \leq r$

Consider:

$$(h_1 \cap h_2)(q) = h_1(q) \wedge (h_2)(q) \geq h_1(r) \wedge h_2(r) = (h_1 \cap h_2)(r).$$

$\Rightarrow h_1 \cap h_2$ is a fuzzy POTΓSG of T.

Theorem 5.5: "The intersection of arbitrary family of fuzzy POTΓSSGs of T is a fuzzy POTΓSSG of T".

Proof: Let h_1 , h_2 , h_3, h_4------ be the family of fuzzy POTΓSGs of T.

1) Consider

$$(\bigcap_{i \in I} h_i)(q\Gamma r\Gamma s) = h_1(q\Gamma r\Gamma s) \wedge h_2(q\Gamma r\Gamma s) \wedge ----$$

$$\geq h_1(q) \wedge h_1(r) \wedge h_2(q) \wedge h_2(r) \wedge h_1(s) \wedge h_2(s) ----$$

$$\geq h_1(q) \wedge h_2(q) \wedge h_1(s) \wedge h_1(r) \wedge h_2(r) \wedge h_2(s)$$

$$\geq (\bigcap_{i \in I} h_i)(q) \wedge (\bigcap_{i \in I} h_i)(r) \wedge (\bigcap_{i \in I} h_i)(s)$$

2) let $q \leq r$

Consider

$$(\bigcap_{i \in I} h_i)(r) = h_1(r) \wedge h_2(r) ---- \geq$$

$$h_1(s) \wedge h_2(s) ---- = (\bigcap_{i \in I} h_i)(s)$$

$\therefore$ The intersection of arbitrary family of fuzzy POTΓSSGs of T is a fuzzy POTΓSSG of T.

Definition 5.6: Let *h* be a FS of a POTΓSG T. The smallest fuzzy POTΓSG of T containing *h* is known as fuzzy POTΓSG of T generated by *h* and is denoted as (h).

Theorem 5.7: Let *h* be a FS of a POTΓSGT. Then (*h*) = The intersection of all fuzzy POTΓSG s of T containing *h*.

Proof: Let $= \{g \,/\, g$ is a fuzzy Po Γ semi group of T and $h \subseteq g\}$

since T itself is a fuzzy POTΓSG and $h \subseteq T$
$\Rightarrow T \in \Delta \Rightarrow \Delta \neq \varnothing$

Let $H^* = \bigcap_{g \in \Delta} g_1 \Rightarrow H^* \neq \varnothing$ by above theorem, H^* is a fuzzy POTΓSG of T.

Since $H^* \subseteq g_1, \forall g_1 \in \Delta, H^*$ is the smallest fuzzy POTΓSG of T containing h.

Hence $H^* = (h)$.

6. Conclusion

The study of fuzzy PO Ternary Γ Sub Semi Group of POTΓSG T, we introduced the notions of FTΓSSG, fuzzy simple POTΓSG, fuzzy identity and fuzzy zero of T. Also showed some more relations between them. Hopefully, some more new results in this topic shall be obtained in the upcoming papers.

References

[1] Anjaneyulu A., Structure and ideal theory of semi groups – Thesis, ANU (1980).

[2] Clifford A.H and Preston G.B., The algebraic theory of semi groups vol – I (American Math. Society, Province (1961)).

[3] Clifford A.H and Preston G.B., The algebraic theory of semi groups vol – II (American Math. Society, Province (1967)).

[4] G. Mohanraj, D. Krishna Swamy, R. Hema, On fuzzy m-systems and n-systems of ordered semi group, Annals of Fuzzy Mathematics and Informatics, Volume X, Number X,2013.

[5] J. N. Mordeson, D. S. Malik, N. Kuroki, Fuzzy Semigroups, Springer-Verlag Berlin Heidelberg Gmbh, 2003(E.Book)

[6] L. A. Zadeh, Fuzzy Sets, Inform. Control.,8(1965) 338-353.

[7] N. Kehayopulu, M. Tsingelis, Fuzzy Sets in Ordered GrouPOids, Semigroup forum 65(2002) 128-132.

[8] N. Kehayopulu, M. Tsingelis, On weakly Prime ideals of ordered Semigroups, Math. Japan. 35(1990) 1051-1056.

[9] N. Kehayopulu, On Prime, weakly prime ideals in ordered semigroups, Semigroup Forum 44(1992) 341-346.

[10] Padmalatha and A. Gangadhara Rao, Anjaneyulu A., PO Ideals in partially ordered semi groups, International Research Journal of Pure Algebra-4(6),2014.

[11] P.M. Padmalatha and A. Gangadhara Rao, Simple partially ordered semi groups, Global Journal of Pure and Applied Mathematics, Volume 10, Number 3(2014).

[12] Xiang-Yun Xie, Jian Tang, Prime fuzzy radicals and fuzzy ideals of ordered Semi groups, Information Sciences 178 (2008), 4357– 4374.

[13] X. Y. Xie, Fuzzy ideals in Semi groups, J. Fuzzy math.,7(1999)357-365.

[14] X. Y. Xie, On prime fuzzy ideals of a Semi groups, J. Fuzzy math.,8(2000)231-241.

[15] V. Siva Ramireddy, “Studied on ideals in partial ordered ternary semi groups.”

[16] J. M. Pradeep, A. Gangadhara Rao, P. Ramyalatha, L. Achala, “Completely Prime Fuzzy, Prime Fuzzy Ideal of a PO Ternary Semi Group.”

[17] S. Chandra, A. Singh and P. K. “Physico chemical, “Analysis of Water from Various Sources and Their Comparative Studies.” (Jul. - Aug. 2013).