

Goal Programming in Simplex Method

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Abstract: In this paper, the definition of goal programming and concept of goal programming is given. Some of its properties are discussed here. Preambles and verified by suitable examples. Every management has the task of accomplishing many financial goals such as dividend payout policy, growth of earning over certain planning and capital structure. This study presents a financial planning to attain such incompatible and incommensurable goals using goal programming. Increasing the both capital structure and growth in earnings are the goals of this study.

Keywords: Goal programming, Definition, Formulation.

1. Introduction

Linear programming basically is the technique applicable only when there is a single goal, such as maximizing the profit or minimizing the cost or loss. There are situations where the system may have multiple goals. For example, maximization of profit, minimizing overtime or cost, etc. In such situations, we need a different technique that seek a compromise solution based on the relative importance of each objective. This technique is known as Goal Programming. Goal programming technique starts with the most important goal and continues until the achievement of a less important goal. Whether the goals are attainable or not, the objective function is stated in such a manner that optimization means: "as close as possible to the indicated goals".

2. Linear Goal Programming Problem

Following are the major steps in the formulation of linear goal programming problem:

Step 1: Identify the decision variables of the key decision.

Step 2: Formulate all the objectives or goals of the problem.

These are generally determined by (i) the desire of the decision maker, (ii) limited resources, (iii) any other restriction either explicitly or implicitly placed on the desire of the decision maker.

Step 3: Reduce the number of goals by eliminating a few negligibly important or redundant goals.

Step 4: Express each goal in the form of constraint equation by introducing a negative and positive deviation variable (denoted by d_i^- and d_i^+ respectively) i.e., $G_i: f_i(x_1, x_2, \dots, x_n) - d_i^- + d_i^+ = b_i; i=1, 2, \dots, m$ Where d_i^- = negative deviation from i^{th} goal (underachievement) and d_i^+ = positive deviation from i^{th} goal (overachievement).

Step 5: Assign the goals to priority levels all absolute goals (i.e., either $d_i^- = 0$ or $d_i^+ = 0$ if any exits are assigned to top priority.

Step 6: Establish the achievement functions. The priority associated with each objective along with the deviation variables are used to form what we call the achievement function.

3. Problem

A firm produces two products, say X and Y. product X sells for a net profit of Rs. 80 per unit, while product Y sells for a net profit of Rs. 40 per unit. The goal of the firm is to earn Rs.900 in the next week. Also, the management want to achieve sales volume for the two products close to 17 and 15 respectively. Formulate this problem as a goal programming model.

A. Formulation

Let x_1 and x_2 denote the number of units of product X and Y respectively. The linear programming formulation of the problem is:

$$\text{Max } z = 80x_1 + 40x_2$$

Subject to the constraints

$$x_1 \leq 17$$

$$x_2 \leq 15 \text{ and } x_1, x_2 \geq 0$$

Since the goal of the firm pertains to profit attainment with a target established at Rs. 900 per week, the constraints of the problem can be stated as:

$$80x_1 + 40x_2 = 900$$

Subject to the constraints

$$x_1 \leq 17 \text{ and } x_2 \leq 15 \text{ and } x_1, x_2 \geq 0$$

The problem can now be formulated as goal programming model as follows:

$$\text{Min } z = d_1^- + d_1^+ + d_2^- + d_2^+$$

Subject to the constraints

$$80x_1 + 40x_2 + d_1^- - d_1^+ = 900$$

$$x_1 + d_2^- = 17$$

$$x_2 + d_3^- = 15$$

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_3^- \geq 0$$

Where d_2^- and d_3^- represent under achievements of sales volume for products A and B respectively. Since sales target goals are given as the maximum possible sales volume, therefore d_2^- and d_3^- are not included in the sales target constraints.

Delton Electronics manufactures two types of TV Sets. One TV set, the Deluxe, requires 2 hours in assembly, while the other, the Supreme, requires 4 hours assembly time. The normal assembly operation is limited to 80 hours per week. Marketing surveys indicate that no more than 60 Deluxe and 30 Supreme TV sets should be produced each week. The net profit from the Deluxe model is Rs. 100 each and is Rs. 150 each from the Supreme model. The company president has stated the following objectives in order of priority.

1. Maximize total profit.
 2. Minimize overtime operation of the assembly line.
 3. Sell as many TV sets as possible
- (this is not necessarily the same as maximizing profit)

Since the profit from the Supreme model is 2 times that from the Deluxe model, the president has 2 times as much desire to maximize the sales of the supreme model as he does for the Deluxe model.

Formulate this as a linear goal programming problem.

Formulation:

The decision variables are:

X_1 = number of Deluxe TV sets built each week'

X_2 = number of Supreme TV sets built each week'

The highest priority objective of the president is to maximize profit. Profit, in turn, can be written as a function of x_1 and x_2 as follows:

$$\text{Profit/week} = 100x_1 + 150x_2$$

Setting profit to an arbitrarily high level of Rs. 5,000 per week, the goal is written as:

$$100x_1 + 150x_2 + d_1^- - d_1^+ = 5,000$$

Note that d_1^- is the negative deviation measure, i.e., the amount by which we underachieve our objective. On the other hand, d_1^+ is the amount by which we overachieve our target profit level.

The second priority objective is the minimization of assembly line overtime operation.

Since assembly line time is normally 80 hours per week, our goal is:

$$2x_1 + 4x_2 + d_2^- - d_2^+ = 80$$

Here d_2^- is the amount of "slack" time on the assembly line while d_2^+ represents the amount of "overtime". Since our objective is measured by the minimizations d_2^+ of overtime, we achieve this goal by minimizing. The third, and final priority is associated with a sales objective. Since marketing has indicated that the demands for Deluxe and Supreme TV sets are 60 and 30 units, respectively per week, we naturally strive to satisfy these demands and our goal is:

$$x_1 + d_3^- - d_3^+ = 60 \text{ and } x_2 + d_4^- - d_4^+ = 30$$

Our third goal is achieved by minimizing d_3^- and fourth goal is achieved by minimizing d_4^- further, since the supreme profit is 2 times the Deluxe profit, we shall have 2 times more desire to minimize d_4^- as to minimize d_3^- ,

The achievement function for the problem then is to find X_1 and X_2 so as to minimize $Z = \{d_3^+ + d_4^+, d_1^-, d_2^+, d_3^-, d_4^-\}$

The final decision model a linear goal programming problem is thus:

Find x_1 and x_2 so as to,

$$\text{Minimize } z = \{d_1^-, d_2^+, d_3^+ + d_4^+, d_3^-, 2d_4^-\}$$

Subject to:

$$100x_1 + 150x_2 + d_1^- - d_1^+ = 5,000,$$

$$2x_1 + 4x_2 + d_2^- - d_2^+ = 80,$$

$$x_1 + d_3^- - d_3^+ = 60, x_2 + d_4^- - d_4^+ = 30$$

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_4^-, d_4^+ \geq 0.$$

4. Conclusion

A new approach for solving goal programming in simplex method is developed, alongside with modification of the existing ones. Difference structure of simplex goal programming and solved using the new approach.

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