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Simplex Method for Goal Programming Problems

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***Abstract*: In this paper, the definition of simplex technique in goal programming issues and idea of simplex technique and a few of its properties and applications area unit mentioned here, definition and algorithms of simplex technique for goal programming issues also are outlined and verified by appropriate examples and issues. Its associate approach used for finding a multi objective improvement downside. The process results for comparatively little issues show that the simplex technique produces an equivalent variety of iterations as different ways. Simplex technique may be accustomed solve issues with quite 2 real variables.**

***Keywords*: Decision variables, deviational variables, key decision, initial basic feasible solution, key column, key row, labour constraint, material constraints, goals.**

1. Introduction

Linear programming essentially is that the technique applicable only if there's one goal (objective function), like increasing the profit or minimizing the price or loss. There are things wherever the system might have multiple (possibly conflicting) goals. for instance, a firm might have a collection of goals, like employment stability, high product quality, maximization of profit, minimizing overtime or value. etc. In such things, we want a distinct technique that look for a compromise answer supported the relative importance of every objective. this method is understood as Goal Programming. It aims at minimizing the deviations from the targets that were set by the management. during this technique, we tend to begin with the foremost vital goal and continues till the accomplishment of a slighter goal. whether or not the goals are possible or not, the target operate is explicit in such a fashion that improvement suggests that "as close as possible to the indicated goals".

Illustration. Consider the following L.P.P.

Maximize $z = x_1 + x_2$.

Subject to the constraints:

$3x_1 + 2x_2 \leq 12, x_1 + x_2 \geq 8, -x_1 + x_2 \geq 4, 5x_1 \leq 10;$

and

$$x_1 \geq 0, x_2 \geq 0.$$

2. Simplex Method for Goal Programming Problem

The major steps of the simplex method for the linear goal programming problem
are:

Step 1: Identify the decision variables of the key decision and formulate the given problem as linear goal programming problem.

Step 2: Determine the initial basic feasible solution and set up initial simplex table. Compute z_j and $z_j - c_j$ values separately for each of the ranked goals $P_1, P_2, \ldots$ and enter at the bottom of the simplex table. These are shown from bottom to top, i.e., first priority goal (P_1) is shown at the bottom and least priority goal at the top.

Step 3: Examine $z_j - c_j$ values in the P_1-row first. If all $(z_j - c_j) \leq 0$ at the highest priority levels, then the optimum solution has been obtained. If at least one $z_j - c_j > 0$ at a certain priority level and there is no negative entry at the higher unachieved priority levels, in the same column, then the current solution is not optimum.

Step 4: If the target values of each goal in the solution column (X_B) is zero, the current solution is optimum.

Step 5: Examine the positive values of $(z_j - c_j)$ of the highest priority (P_1) and

Choose the largest of these. The column corresponding to this value becomes the key column. Otherwise move to the next higher priority (P_2) and select the largest positive value of $(z_j - c_j)$ for determining the key column.

Step 6: Determine the key row and key number (leading element) in the same way as in the Simplex Method.

Step 7: Any positive value in the $(z_j - c_j)$ row which has negative $(z_j - c_j)$ under any lower priority rows are ignored. It is because deviations from highest priority goal would be increased with the entry of this variable in the basis.

Problem:

The production manager of a company wants to schedule a week's production run for two products A and B, each of which requires the labour and materials as shown in table 1.

The unit profit for A and B is Rs. 20 and Rs. 2 respectively.

The manager would like to maximise profit, but he is equally concerned with maintaining workforce of the division at nearly constant level in the intersect of employee morale. The work, which consists of people engaged in production, sales,

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distribution and other general staff is consisted of 108 persons in all. Also, it is known that the production of one unit of A would maintain 0.3 persons in the workforce and one unit of B would maintain 0.75 persons.

Table 1

Product	**Labour (in hours)**	**Material M1 (in kgs.)**	**Material M2 (in kgs.)**
A	2	4	5
B	4	5	4
Available (per week)	600	1,000	1,200

Had the production manager been considering only maximising profit, without regard to maintaining the workforce, he would do so by producing 167.67 units of A and 66.67 units of B. On the basis of the available capacity, this would yield a profit of Rs. 5,486.67. However, this would maintain 100.3 people in the workforce. The manager feels that probably he could increase the workforce requirement, o the desired level by accepting somewhat lower profit. So, the following two goals are to be achieved:

(a) profit of Rs.5,400 per week and (b) workforce of 108 persons.

Formulate and solve the given problem as linear goal programming problem.

Formulation:

Let x_1= number of units of product A to be produced every week and x_2= number of units of product B to be produced every week. Then, the constraints and goals of the problem can be expressed as follows:

$2x_1 + 4x_2 \leq 600$ (Labour Constraint)
$4x_1 + 5x_2 \leq 1{,}000$ (Material M_1Constraint)
$5x_1 + 4x_2 \leq 1{,}200$ (Material M_2 Constraint)
$20x_1 + 32x_2 = 5{,}400$ (Goal 1)
$0.3x_1 + 0.75x_2 = 108$ (Goal 2)

Since, we have to satisfy goal 1 and goal 2 simultaneously, the given problem may not have feasible solution. Further, in order to solve the given problem by simplex method, we introduce the deviational variables d_1^+ andd_1^- in goal 1 constraint and, d_2^+ and d_2^- in goal 2 constraints, where,

d_1^+= number of rupees above the goal of Rs. 5,400,
d_1^-= number of rupees below the goal of Rs. 5,400,
d_2^+= number of people above the workforce goal of 108,
d_2^-= number of people below the workforce goal of 108.

Making use of slack variables $s_1 \geq 0, s_2 \geq 0, s_3 \geq 0$ in the first three constraints respectively, and the deviational variables in the fourth and fifth constraints; the goal linear programming problem is,

Minimize $z = d_1^- + d_2^- + 0s_1 + 0s_2 + 0s_3 + 0d_1^+ + 0d_2^+$

Subject to the constraints:

$$2x_1 + 4x_2 + s_1 = 600,$$
$$4x_1 + 5x_2 + s_2 = 1{,}000,$$
$$5x_1 + 4x_2 + s_3 = 1{,}200,$$
$$20x_1 + 32x_2 + d_1^- - d_1^+ = 5{,}400,$$
$$0.3x_1 + 0.75x_2 + d_2^- + d_2^+ = 108,$$

$x_1, x_2, s_1, s_2, s_3 \geq 0 \; and \; d_1^-, d_1^+, d_2^-, d_2^+ \geq 0$ Solution by Simplex Method.

Using simplex method an initial (starting) basic feasible solution is,

$s_1 = 600, s_2 = 1{,}000, s_3 = 1{,}200, d_1^- = 5{,}400, d_1^+ = 108.$

With I_5 as the initial basis matrix.

Initial Iteration:

Introduce y_2 and dropd_2^-.

Since, $z_1 - c_1 > 0 \; and \; z_2 - c_2 > 0$, current solution is not optimum. As the largest of these two positive quantities is 32.75 correspondingto $z_2 - c_2$, $\boldsymbol{y_2}$ enters the basis. Further,

$Min.\left\{\frac{x_{Bi}}{y_{i2}}, y_{i2} > 0\right\}$=$min.\left\{\frac{600}{4}, \frac{1000}{5}, \frac{1200}{4}, \frac{5400}{32}, \frac{108}{0.75}\right\} = \frac{108}{0.75}$.

This implies that $\boldsymbol{d_2^-}$ leaves the basis.

First Iteration:

Introduce $\boldsymbol{d_2^+}$ and drop $\boldsymbol{y_3}$.

			0	0	0	0	0	1	0	1	0
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	d_1^-	d_1^+	d_2^-	d_2^+
0	y_3	600	2	4	1	0	0	0	0	0	0
0	y_4	1,000	4	5	0	1	0	0	0	0	0
0	y_5	1,200	5	4	0	0	1	0	0	0	0
1	d_1^-	5,400	20	32	0	0	0	1	-1	0	0
1	d_2^-	108	0.3	**0.75**	0	0	0	0	0	1	-1
	Z=	5508	20.3	32.75	0	0	0	0	-1	0	-1

			0	0	0	0	0	1	0	1	0
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	d_1^-	d_1^+	d_2^-	d_2^+
0	y_3	24	2/5	0	1	0	0	0	0	-16/3	**16/3**
0	y_4	280	2	0	0	1	0	0	0	-20/3	20/3
0	y_5	624	17/5	0	0	0	1	0	0	-16/3	16/3
1	d_1^-	792	36/5	0	0	0	0	1	-1	-128/3	128/3
0	y_2	144	2/5	1	0	0	0	0	0	4/3	-4/3
	Z =	792	36/5	0	0	0	0	0	-1	-131/3	128/3

			0	0	0	0	0	1	0	1	0
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	d_1^-	d_1^+	d_2^-	d_2^+
0	d_2^+	9/2	**3/40**	0	3/16	0	0	0	0	-1	1
0	y_4	250	3/2	0	-5/4	1	0	0	0	0	0
0	y_5	600	3	0	-1	0	1	0	0	0	0
1	d_1^-	600	4	0	-8	0	0	1	-1	0	0
0	y_2	150	1/2	1	1/4	0	0	0	0	0	0
	Z =	600	4	0	-8	0	0	0	-1	0	0

			0	0	0	0	0	1	0	1	0
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	d_1^-	d_1^+	d_2^-	d_2^+
0	y_1	60	1	0	5/2	0	0	0	0	-40/3	40/3
0	y_4	160	0	0	-5	1	0	0	0	20	-20
0	y_5	420	0	0	-17/2	0	1	0	0	40	-40
1	d_1^-	360	0	0	-18	0	0	1	-1	**160/3**	-160/3
0	y_2	120	0	1	-1	0	0	0	0	20/3	-20/3
	Z =	360	0	0	-18	0	0	0	-1	157/3	-160/3

			0	0	0	0	0	1	0	1	0
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	d_1^-	d_1^+	d_2^-	d_2^+
0	y_1	150	1	0	-2	0	0	1/4	-1/4	0	0
0	y_4	25	0	0	7/4	1	0	-3/8	3/8	0	0
0	y_5	150	0	0	5	0	1	-3/4	3/4	0	0
1	d_2^-	27/4	0	0	-27/80	0	0	3/160	-3/160	1	-1
0	y_2	75	0	1	5/4	0	0	-1/8	1/8	0	0
	Z =	27/4	0	0	-27/80	0	0	-157/160	-3/160	0	-1

Since, $z_9 - c_9 = \frac{128}{3}$ is largest positive net evaluation, d_2^+ enters the basis, Further,

$Min.\left\{\frac{x_{Bi}}{y_{i9}}, y_{i9} > 0\right\}$ $is \frac{24}{\frac{16}{3}}$. This implies $\boldsymbol{y_3}$ leaves the basis.

Second Iteration:

Introduce $\boldsymbol{y_1}$ and drop $\boldsymbol{d_2^+}$.

Clearly, $\boldsymbol{y_1}$ enters the basis, since $z_1 - c_1 > 0$. Also,

$Min.\left\{\frac{x_{Bi}}{y_{i1}}, y_{i1} > 0\right\} = \frac{\frac{9}{2}}{3}/40$ indicates $\boldsymbol{d_2^+}$ leaves the basis.

Third Iteration:

Introduce $\boldsymbol{d_2^-}$ and drop $\boldsymbol{d_1^-}$.

Clearly, $\boldsymbol{d_2^-}$ enters the basis because $z_8 - c_8 > 0$, and $\boldsymbol{d_1^-}$ leaves the basis.

Final Iteration:

Optimum Solution.

Since, all $z_j - c_j \leq 0$, an optimum solution is obtained. Hence, the optimum solution is:

$x_1 = 150, x_2 = 75, d_2^- = \frac{27}{4} = 6.75, s_2 = 25\ and\ s_3 = 150$ with the minimum of z = 6.75.

This implies that the workforce shall be 108-6.75(=101.25), with the employment goal being under-achieved to the extent of 6.75 people; while 25 kg. of material M_1 and 150 kg. of material M_2 would remain unutilised. The other variables are non-basic so that all the available labour hours shall be used and the profit goal be met exactly.

3. Conclusion

Goal programming has a great deal of flexibility that is lacking in linear programming. Furthermore, the approach of multiple goal attainment, according to their priorities, is readily suitable to most management decision-problems.

References

[1] Gamal, Moh Danil. (2015). Solving Linear Goal Programming Using Dual Simplex Method. Bulletin of Mathematics. 7. 1-9.