

Fuzzy Linear Mathematical Programme using in Healthcare Analysis

K. Kowsalya^{1*}, P. Keerthana², R. Bharathi³

^{1,2}Student, Department of Mathematics, STET Women's College (Autonomous), Mannargudi, India

³Assistant Professor, Department of Mathematics, STET Women's College (Autonomous), Mannargudi, India

Abstract: Most countries in India have a limited number of healthcare facilities per capita. The excess of Indian population has been revealed to cause a number of contagious diseases, especially in diseases, nutritional facilities in this paper.

Keywords: Healthcare facilities, contagious diseases, linear programming problem, multi linear model.

1. Introduction

In most growing and growing countries, especially in India, there are a limited number of health service protective measures, mainly medicines, per capita. The Indian population was unprotected until a number of infectious diseases, nutritional issues and medical care facilities. Different from allopathic medicine, different forms of scientifically sound and tolerable systems of aboriginal medicine, such as herbalism, Unani, Siddha and homeopathy system, are practiced in different parts of Indian nation. Support the systems of drugs to counteract the provocation of the lack of health facilities and to build resources promotion in healthcare organization. In recent years there has been a dramatic increase in the application of optimization techniques to the study of computer science health facilities and to strengthen to the health care system to indicate the wide distribution range of some typical application in health analysis are listed below.

Definition 1.1:

Let R be the set of all real numbers. We assume that a fuzzy number A for all $x \in R$ can be expressed in the form,

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}_L}(x) & a \leq x \leq b \\ W & b \leq x \leq c \\ \mu_{\tilde{A}_R}(x) & c \leq x \leq d \\ 0 & \text{Otherwise} \end{cases}$$

Where $0 \leq w \leq 1$ is a constant, a, b, c, d are real numbers, such that $a < b \leq c < d$, $\mu_{\tilde{A}_L}(x): [a, b] \rightarrow [0, w]$, $\mu_{\tilde{A}_R}(x): [c, d] \rightarrow [0, w]$ are two strictly monotonic and continuous functions from R to the close interval $[0, w]$.

Since $\mu_{\tilde{A}_L}(x)$ is continuous and strictly increasing, the inverse

function can be denoted by a $\mu_{\tilde{A}_R}(X)$ is exists. Similarly, $\mu_{\tilde{A}_R}(x)$ is continuous and strictly decreasing, the inverse function of $\mu_{\tilde{A}_R}(x)$ also exist. The inverse functions of $\mu_{\tilde{A}_L}(x)$ and $\mu_{\tilde{A}_R}(x)$ can be denoted by $\mu_{\tilde{A}_L}^{-1}(x)$ and $\mu_{\tilde{A}_R}^{-1}(x)$, respectively. $\mu_{\tilde{A}_L}^{-1}(x)$ and $\mu_{\tilde{A}_R}^{-1}(x)$ are continuous on $[0, w]$ that means both $\int_0^w \mu_{\tilde{A}_L}^{-1}(x)$ and $\int_0^w \mu_{\tilde{A}_R}^{-1}(x)$ exist.

Definition 1.2:

A fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} W \left(\frac{x-a}{b-a} \right) & a \leq x \leq b \\ W & b \leq x \leq c \\ W \left(\frac{x-d}{c-d} \right) & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

Definition 1.3:

A fuzzy number $\tilde{A} = (a, b, c; w)$ is said to be triangular generalized triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq a \\ W \left(\frac{x-a}{b-a} \right) & a \leq x \leq b \\ W \left(\frac{c-x}{c-b} \right) & b \leq x \leq c \\ 0 & x > c \end{cases}$$

A. Proposed linear model with multiple objectives in controlling communicable diseases

In this section, a Fuzzy Multi Objective Linear Programming Model is described proposed for the calculation based on a multi objective fuzzy transport model minimal treatment cost as and healing time of an affected diseases population various communicable diseases to minimize human protective loss.

This model is concerned with determining the minimum total cost of treatment and healing time of a disease population affected by various communicable diseases to be healed by various treatments in a region. The data of the model include,

*Corresponding author: kk1533224@gmail.com

- The size of patients affected by each disease to be treated and managed the overall availability of different treatments in a given region.
- The treatment cost and curing time per unit (i.e., treatment cost and curing time per patients) of the disease.

Table 1
Fuzzy model for optimization of cost and time of treatment

Treatments/Diseases	B1	B2	Bj	Bn	Supply (availability of treatment Bj)
A1	\sim $\tilde{c}_{11}; \tilde{t}_{11}; \tilde{d}_{11}$	\sim $\tilde{c}_{12}; \tilde{t}_{12}; \tilde{d}_{12}$	\sim $\tilde{c}_{1j}; \tilde{t}_{1j}; \tilde{d}_{1j}$	\sim $\tilde{c}_{1n}; \tilde{t}_{1n}; \tilde{d}_{1n}$	\tilde{a}_1
A2	\sim $\tilde{c}_{21}; \tilde{t}_{21}; \tilde{d}_{21}$	\sim $\tilde{c}_{22}; \tilde{t}_{22}; \tilde{d}_{22}$	\sim $\tilde{c}_{2j}; \tilde{t}_{2j}; \tilde{d}_{2j}$	\sim $\tilde{c}_{2n}; \tilde{t}_{2n}; \tilde{d}_{2n}$	\tilde{a}_2
Ai	\sim $\tilde{c}_{i1}; \tilde{t}_{i1}; \tilde{d}_{i1}$	\sim $\tilde{c}_{i2}; \tilde{t}_{i2}; \tilde{d}_{i2}$	\sim $\tilde{c}_{ij}; \tilde{t}_{ij}; \tilde{d}_{ij}$	\sim $\tilde{c}_{in}; \tilde{t}_{in}; \tilde{d}_{in}$	\tilde{a}_i
: ⋮			: ⋮		: ⋮		: ⋮
Am	\sim $\tilde{c}_{m1}; \tilde{t}_{m1}; \tilde{d}_{m1}$	\sim $\tilde{c}_{m2}; \tilde{t}_{m2}; \tilde{d}_{m2}$	\sim $\tilde{c}_{mj}; \tilde{t}_{mj}; \tilde{d}_{mj}$	\sim $\tilde{c}_{mn}; \tilde{t}_{mn}; \tilde{d}_{mn}$	\tilde{a}_m
Demand	\tilde{b}_1	\tilde{b}_2	\tilde{b}_j	\tilde{b}_n	

Table 2
Treatment cost & time per patient

Treatment	Disease	Treatment Cost per Patient (in Rupees)	Curing Time per Patient (in days)	Dosage per Patient (in gms) (per course)
Allopathy	Typhoid	(2700,3500,3600)	(31,35,37)	(5, 6, 7)
	Jaundice	(2100,2500,2800)	(20,22,25)	(8, 10, 12)
	Pneumonia	(8300,8500,9100)	(250,270,300)	(450, 500, 550)
Ayurvedic	Typhoid	(1700,2000,2300)	(19,20,24)	(100, 300, 500)
	Jaundice	(2900,3200,3400)	(29,30,34)	(500, 550, 600)
	Pneumonia	(4600,4800,5100)	(90,120,130)	(1000,1500,2000)
Homeopathy	Typhoid	(4000,4200,4500)	(70,90,110)	(0.5, 0.7, 1.0)
	Jaundice	(4400,4800,5000)	(55,75,95)	(1, 1.5, 2)
	Pneumonia	(3700,4100,4300)	(325,345,355)	(40, 45, 50)
Unani	Typhoid	(3500,3800,4000)	(100,120,130)	(200, 300, 400)
	Jaundice	(3800,4100,4400)	(60,90,120)	(400, 470, 500)
	Pneumonia	(5300,5500,5700)	(400,420,450)	(1000, 1250, 1500)

Table 3
Unbalanced table with fuzzy treatment cost and fuzzy curing time

Treatments/Diseases	Typhoid (B1)	Jaundice (B2)	Pneumonia (D3)	Supply (availability of treatment Tj)
Allopathy (A1)	(2700,3500,3600) (31,35,37) (5, 6, 7)	(2100,2500,2800) (20,22,25) (8, 10, 12)	(8300,8500,9100) (250,270,300) (450, 500, 550)	(50000,52000, 55000)
Ayurvedic (A2)	(1700,2000,2300) (19,20,24) (100, 300, 500)	(2900,3200,3400) (29,30,34) (500, 550, 600)	(4600,4800,5100) (90,120,130) (1000, 1500, 2000)	(31000,34000, 37000)
Homeopathy (A3)	(4000,4200,4500) (70,90,110) (0.5, 0.7, 1.0)	(4400,4800,5000) (55,75,95) (1, 1.5, 2)	(3700,4100,4300) (325,345,355) (40, 45, 50)	(10500,12500, 14500)
Unani (A4)	(3500,3800,4000) (100,120,130) (200, 300, 400)	(3800,4100,4400) (60,90,120) (400, 470, 500)	(5300,5500,5700) (400,420,450) (1000, 1250, 1500)	(5500,7500, 9500)
Demand				
(no. of patients affected disease)	(21500,22500,2500)	(14250,17250,19500)	(10250,12450,15500)	

Table 4

Treatments/Diseases	Dengue (B1)	Jaundice (B2)	Tuberculosis (B3)	(B)	Supply (Availability of treatment Bj)
Allopathy (A1)	3344	2478	8589	0	52222
	35	22	272	0	
	6	10	500	0	
Ayurvedic (A2)	2000	3178	4822	0	34000
	21	31	116	0	
	300	550	1500	0	
Homeopathy (A3)	4222	4756	4056	0	12500
	90	75	343	0	
	0.7	1.5	45	0	
Unani (A4)	3778	4100	5500	0	7500
	118	90	422	0	
	300	461	1250	0	
Demand (No. of patients affected by the disease Bj to be taken the treatment)	22944	17083	12639	53556	106222

The aims to be determine how the different treatments can be distributed among the different disease populations to minimize the total treatment cost as and to minimize curing time. Hence, the decision variables are:

X_{ij} = the affordability of the j^{th} treatment for the i^{th} disease, where $i=1,2,3,\dots,m$ and $j= 1,2,3,\dots,n$.

That is a set of $m \times n$ variables.

In order to minimize the treatment cost and time, the following problem needs to be solved.

B. The Objective Function

Consider the size of patients to be treated i who are affected by a disease j . For all i and all those unit treatment costs are c_{ij} , unit curing time t_{ij} , unit dosage d_{ij} , affordability of treating disease x_{ij} . As we assume that cost and time functions are linear, the sum treatment costs, total curing time and total dosage is given by $c_{ij}x_{ij}$, $t_{ij}x_{ij}$ and $d_{ij}x_{ij}$ respectively. Summation over all i and all j now gives the total treatment costs, curing time and dosage for all disease – treatment combinations. That’s, our goals functions are,

$$\begin{aligned} \text{Minimize } \tilde{z} &= \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} x_{ij} \\ \text{Minimize } \tilde{z} &= \sum_{i=1}^m \sum_{j=1}^n \tilde{t}_{ij} x_{ij} \\ \text{Minimize } \tilde{z} &= \sum_{i=1}^m \sum_{j=1}^n \tilde{d}_{ij} x_{ij} \end{aligned}$$

Then it is a transport with two destinations with the weights of the goals that take into account the priorities of the goal.

$$\tilde{Z} = w_1 \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} x_{ij} + w_2 \sum_{i=1}^m \sum_{j=1}^n \tilde{t}_{ij} x_{ij} + w_3 \sum_{i=1}^m \sum_{j=1}^n \tilde{d}_{ij} x_{ij}$$

C. The restrictions

Consider treatment i , the overall affordability of that treatment for the various given diseases in the region is the sum $x_{i1} + x_{i2} + \dots + x_{in}$. Since the availability of this treatment for various diseases in the region is a_i , the affordability of this treatment for the various given diseases cannot exceed a_i .

$$\text{(i.e.) } \sum_{j=1}^n x_{ij} \leq a_i \text{ For } i= 1, 2, \dots, m$$

Consider disease j . think of sickness as the sum total of affordability of different circumstances treatments for this disease in the region total is $x_{1j} + x_{2j} + \dots + x_{mj}$. Because the overall size of patients affected disease are taken treatment is b_j , the overall affordability of different treatments should not be less than b_j .

$$\text{(i.e.) } \sum_{i=1}^m x_{ij} \leq b_j \text{ for } j= 1,2,\dots,n$$

Where $x_{ij} \geq 0$ for all i and j

The above implies that the total availability of different is given treatments for various given diseases $\sum_{i=1}^m a_i$ is greater than or equals the total number of patients affected by the various given diseases $\sum_{j=1}^n b_j$ if the total availability of different given treatments is equal to the total number of

patients affected by the various diseases given (i.e. $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$) then the model is said to be balanced. In a balanced model, each of the constraints is an equation.

$$\begin{aligned} \sum_{i=1}^m x_{ij} &= a_i \text{ for } i= 1, 2, \dots, m \\ \sum_{j=1}^n x_{ij} &= b_j \text{ for } j= 1, 2, \dots, n \end{aligned}$$

A model in which total availability of various given treatments is given and total number of patients affected by the various claims. diseases are unequal means unbalanced. It is always possible to balance an unbalanced problem.

The fuzzy problem, where treatment costs c_{ij} , healing time t_{ij} , dosage d_{ij} total availability of treatment a_i and the total number of patients are to treated b_j quantities are fuzzy quantities, can be formulated as follows:

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} x_{ij} + w_2 \sum_{i=1}^m \sum_{j=1}^n \tilde{t}_{ij} x_{ij} + w_3 \sum_{i=1}^m \sum_{j=1}^n \tilde{d}_{ij} x_{ij}$$

$$\text{Subject to } \sum_{j=1}^n x_{ij} \leq \tilde{a}_i \text{ for } i= 1, 2, \dots, m$$

$$\text{And } \sum_{i=1}^m x_{ij} \geq \tilde{b}_j \text{ for } j= 1, 2, \dots, n$$

Where $x_{ij} \geq 0$ for all i and j

This fuzzy problem is shown explicitly in the table 1.

Communicable diseases are diseases that occur as a result of pathogen that spreads from one person to another. They are among the main causes of illnesses in many countries. These diseases affect people of all ages but even more so children due to their expose to environmental conditions that support the dissemination. Communicable diseases are preventable basis interventions at different transmission levels of the illness. Health authorities an important role combating these diseases through effective and efficient use management, preventions and control measures.

In Thiruvavur Region, the availability of various treatments such as Allopathy (A_1), Ayurvedic (A_2), Homeopathy (A_3) and Unani (A_4) for all type of diseases are (50000,52000,55000), (31000, 34000, 37000), (10500, 12500, 14500), and (5500, 7500, 9500) respectively. Moreover, the size of patients affected by the communicable diseases in winter season like Typhoid (B_1), Malaria (B_2) and Pneumonia (B_3) are (21500, 22500, 25500), (14250, 17250, 19500), and (10250, 12450, 15500) respectively. Treatment cost and curing time for all above said treatment - disease combination per patient is given in table 2. The data are collected from the Department of Medical and Rural Health Services at Thiruvavur District.

Let us consider an optimization problem in Table 3 with rows performing treatments Allopathy (A_1), Ayurvedic (A_2), Homeopathy (A_3) and Unani (A_4) and column representing communicable diseases Typhoid (B_1), Jaundice (B_2) and Pneumonia (B_3) which are affected in the winter season at Thiruvavur Region.

Using equate function in rank function equation, the values

of $R(\tilde{c}_{ij})$, $R(\tilde{t}_{ij})$, $R(\tilde{a}_i)$ and $R(\tilde{b}_j)$ for all i and j are calculated and given in table 4. The problem in table 5 is unbalanced. For make it as a balanced one, the dummy column is introduced.

The transport problem with multiple destinations was given converts into the following crisp linear programming problem

$$\text{Minimize } (688.1)y_{11} + (509.6)y_{12} + (2003.8)y_{13} + (0)y_{14} + (500.5)y_{21} + (816.1)y_{22} + (1472.4)y_{23} + (0)y_{24} + (889.61)y_{31} + (989.15)y_{32} + (996.2)y_{33} + (0)y_{34} + (904.6)y_{41} + (1003.3)y_{42} + (1686)y_{43} + (0)y_{44}.$$

Subject to:

$$\begin{aligned} y_{11} + y_{21} + y_{31} + y_{41} &= 22944 \\ y_{12} + y_{22} + y_{32} + y_{42} &= 17083 \\ y_{13} + y_{23} + y_{33} + y_{43} &= 12639 \\ y_{14} + y_{24} + y_{34} + y_{44} &= 53556 \\ y_{11} + y_{12} + y_{13} + x_{14} &= 52222 \\ y_{21} + y_{22} + y_{23} + y_{24} &= 34000 \\ y_{31} + y_{32} + y_{33} + x_{34} &= 12500 \\ y_{41} + y_{42} + y_{43} + y_{44} &= 7500 \end{aligned}$$

Using XL solver, the linear programming problem is solved to find the optimum solution which is as follows:

$$y_{12}=17083, y_{21}=22944, y_{33}=12500, y_{14}=35139, y_{23}=139, y_{24}=10917, y_{44}=7500.$$

The minimum total cost of fuzzy treatment of fuzzy hardening time are obtained as follows:

$$\text{Overall Minimally blurred Repair Cost,} \\ = \text{Rs. } (135434900, 152470800, 165312300)$$

$$\text{Overall Minimum Fuzzy Cure Time} \\ = (5006353, 5300550, 5587048) \text{ days}$$

$$\text{Overall Minimum Dosage} \\ = (11474900, 17049850, 22624800) \text{ gms}$$

After defuzzification, by using the ranking function in eqn. (4), the overall minimum treatment cost and curing time respectively are Rs.151538711, 5298839 days and of total minimum dosage is 17049850 gms.

Defuzzification:

The process of converting the fuzzy output to the value is should to be defuzzification. A set of defuzzification techniques are known, including centre-of-area, centre of gravity, and mean of the maxima. A common and useful defuzzification technique is the focus. This technique was developed by Sugeno in 1985. This is the most commonly used technique and is very accurate.

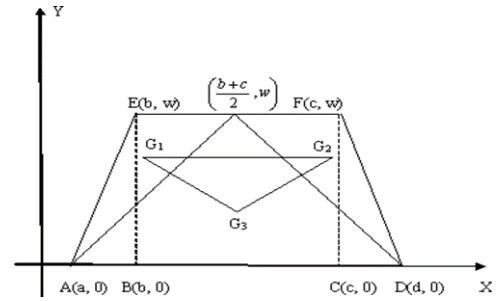


Fig. 1. Focus of focus

We define the centroid $G(\bar{x}_0, \bar{y}_0)$ of the triangle with corners $G_1, G_2,$ and G_3 of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ as

$$G(\bar{x}_0, \bar{y}_0) = \frac{4a+5b+5c+4d}{18}, \frac{5w}{9} \tag{1}$$

Its Ranking function is defined as

$$R(\tilde{A}) = \frac{4a+5b+5c+4d}{18} \tag{2}$$

As a special case, for triangular number $\tilde{A} = (a, b, c, d; w)$, i.e., $c=b$ the focus of focus is given by

$$G(\bar{x}_0, \bar{y}_0) = \frac{2a+5b+2d}{9}, \frac{5w}{9} \tag{3}$$

Its Ranking function is defined by

$$R(\tilde{A}) = \frac{2a+5b+2d}{9} \tag{4}$$

2. Conclusion

This paper presented a study fuzzy linear mathematical programme using in healthcare analysis.

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