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On Cosymplectic Manifold with H-Conformal Curvature $\bar{C}$

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***Abstract*: Tokagi, H and Watanabe [1] Yano, Y. [2], Mishra, R.S. [3], Pandey [4] etc., have studied H-Conformal Curvature tensor $\bar{C}$, The studies of Cosymplectic manifold with orthogonal basis equipped with different structure have been made by Yano [2], Tokagi [1] and Mishra[3].**

Here we have discussed Cosymplectic manifold M_n (n=2m+1) possessing the orthonormal basis {e_i, Fe_i}, i=1, 2, 3-------------2m of unit vector which are normal to the contact vector T, we have obtained the expression relating the sectional curvature and scalar curvature in H-Conformal $\bar{C}$ curvature tensor.

***Keywords*: Almost contact metric (almost Grayan) manifold, Cosymplectic manifold, H-Conformal Curvature tensor, Orthonormal basis, Sectional curvature.**

1. Introduction

Let M_n, n = 2m+1 be an almost contact metric (almost Grayan) manifold equipped with an almost contact metric structure,

{F, T, A, g} satisfying:

(1.1) (a) $F^2 X = - X + A(X) T$

(1.1) (b) $A(FT) = 0$

(1.1)(c) $FT = 0$

(1.1)(d) $A(T) = 0$

(1.2)(a) $g(\bar{X}, \bar{Y}) = g(X,Y) - A(X)A(Y)$

(1.2)(b) $g(T, X) = A(X)$

(1.2)(c) $'F(X, Y) \overset{\text{def}}{=} g(\bar{X}, Y) = - g(X, \bar{Y}) = - 'F(Y, X)$

Where

(1.2)(d) $\bar{X} \overset{\text{def}}{=} FX$,

For all C^∞ vector fields X, Y in M_n, here F is a structure tensor of type (1, 1), A is a 1- form, T is a contravariant vector field associated with A, g is a fundamental metric tensor and ‘F is a fundamental 2- form.

Let D be a Levi - cevita or Riemannian curvature tensor in M_n. If in M_n, the structure tensor F and the contact form A are covariantly constant i.e.

(1.3) $(D_x F)(Y) = 0$

(1.4)(a) $(D_x A)(Y) = 0$

(1.4)(b) $D_x T = 0$

Then M_n is called a Cosymplectic Manifold [2] and [3].

1.40. Ortho-normal basis in M_n:

Let a point $X \in M_n$ {e_1, e_2, e_3,…………e_{2m}, Fe_1, Fe_2…….. Fe_{2m}}, be an orthonormal basis of the tangent space Tx (M_n), such that

(1.40)(a) $K(e_i) = \lambda_i e_i + \mu T$

$K(Fe_i) = \lambda_i Fe_i$, for i = 1,2,3,……….2m.

Where T is such that

(1.40)(b) $g(e_i, T) = 0$,

i.e. T is orthogonal to e_i, for i = 1,2,3…….2m. The result in (1.40) are analogous to those in [1].

Since in cosymplectic manifold M_n (1.3) implies

(1.41)(a) $K(X, Y, \bar{Z}) = \bar{K}(X, Y, Z)$

(1.41)(b) $Ric(Y, \bar{Z}) = Ric(\bar{Y}, Z) = - g(K(\bar{Y}), Z)$

and

(1.41)(c) $K(\bar{Y}) = K(\bar{Y})$

We know that sectional curvature k* of M_n in the plane of the unit vector X and Y at any point $p \in M_n$ is defined by [3].

(1.42) $k^* = (K(X,Y,X,Y))/(g(X,X)g(Y,Y)-\{g(X,Y)\}^2)$

So the sectional curvature of M_n in the plane of e_i, e_j, is given by,

(1.43) $k^* = {'K}(e_j, e_i, e_j, e_i)$

Since $g(e_j, e_i) = 0$, and $g(e_i, e_i) = 1$, as the e_i, e_j are mutually perpendicular.

Now H-conformal $\tilde{C}$ curvature tensor is given by[1],[2],[3]

(2.00) $\widetilde{C}(X,Y,Z,W) \overset{\text{def}}{=} g(\tilde{C}(X,Y,Z),W)$

$= {'K}(X,Y,Z,W) - \frac{1}{(n+4)}\{Ric(Y,Z)g(X,W) - Ric(X,Z)g(Y,W) + Ric(\bar{Y},Z){'F}(X,W) - Ric(\bar{X},Z){'F}(Y,W) + {'F}(Y,Z)Ric(\bar{X},W) - {'F}(X,Z)Ric(\bar{Y},W) + g(Y,Z)Ric(X,W) - g(X,Z)Ric(Y,W) - 2Ric(\bar{X},Y){'F}(Z,W) - 2{'F}(X,Y)Ric(\bar{Z},W)\}$

$+\frac{k}{(n+2)(n+4)}[g(Y,Z)g(X,W) - g(X,Z)g(Y,W) + {'F}(Y,Z){'F}(X,W) - {'F}(X,Z){'F}(Y,W) - 2{'F}(X,Y){'F}(Z,W)]$

Further, from equation (2.00) H-conformal $\tilde{C}$ curvature tensor is given as,

(2.01) $\widetilde{C}(X,Y,Z) = K(X,Y,Z) - \frac{1}{(n+4)}[Ric(Y,Z)X - Ric(X,Z)Y + Ric(\bar{Y},Z)\bar{X} - Ric(\bar{X},Z)\bar{Y} + K(\bar{X})g(\bar{Y},Z) - K(Y)g(X,Z) +$

K(X)g(Y,Z)–K($\bar{Y}$)g($\bar{X}$,Z)–2Ric($\bar{X}$,Y)$\bar{Z}$–2K($\bar{Z}$)g($\bar{X}$,Y)]
+ $\frac{k}{(n+2)(n+4)}$[g(Y,Z)X–g(X,Z)Y+g($\bar{Y}$,Z)$\bar{X}$–g($\bar{X}$,Z)$\bar{Y}$)-2g($\bar{X}$,Y)$\bar{Z}$]

For Z= T, (2.01) becomes,

(2.02) $\widetilde{C}$(X,Y,T) = K(X,Y,T) - $\frac{1}{(n+4)}$[Ric(Y,T)X – Ric(X,T)Y – K(Y)A(X) + K(X)A(Y)]+ $\frac{k}{(n+2)(n+4)}$[A(Y)X – A(X)Y]

Now, putting X = e_i , Y= e_j in above equation ,we get

(2.03) $\widetilde{C}$(e_i ,e_j ,T) = K(e_i ,e_j, T) - $\frac{\mu}{(n+4)}$[e_i - e_j]

Also from (2.02), we get

(2.04) $\widetilde{C}$($\bar{X}$,$\bar{Y}$,T) = K($\bar{X}$,$\bar{Y}$,T)

Again putting X= e_i , Y= e_j in (2.04), we get

(2.05) $\widetilde{C}$(Fe_i ,Fe_j ,T) = K(Fe_i ,Fe_j, T)

Further from (2.03), we obtained

(2.06) $\widetilde{C}$(e_i ,e_j ,e_k, T) = 'K(e_i ,e_j,e_k , T)

Since g(e_i ,e_k) = 0 = g(e_j ,e_k) , $i \neq j \neq k$

Thus, we have,

Theorem(2.10): Let M_n be a cosymplectic manifold .if { e_i ,Fe_j }, i= 1,2,3……….2m; be an orthonormal basis normal to T in M_n , then H-Conformal curvature tensor $\widetilde{C}$ equals the Riemann-Curvature tensor in M_n.

Proof: The proof of the theorem follows immediately from the equation (2.05) and (2.06).

Corollary (2.11): Let M_n be a cosymplectic manifold admitting an orthonormal basis {e_i, Fe_j}, i= 1,2,3……….2m; normal to T . Then H-Conformal curvature tensor $\widetilde{C}$ vanishes. if M_n is flat with respect to this basis.

The proof of the corollary is obvious from the above theorem.

Now, (2.01) gives for X= e_i, Y= e_j

(2.07) $\widetilde{C}$(e_i ,e_j ,Z) = K(e_i ,e_j, Z) - $\frac{1}{(n+4)}$[g(e_j ,Z){$\lambda_j e_i$ +$e_i\lambda_i$ +μT} - g(e_i, Z){ $\lambda_i e_j$ +$e_j\lambda_j$ +μT }+g(Fe_j,Z){ λ_i Fe_i +λ_i Fe_i}-g(Fe_i,Z){ λ_i Fe_j + λ_j Fe_j }-2g(Fe_i ,e_j){ λ_i $\bar{Z}$ +K($\bar{Z}$)}] + $\frac{k}{(n+2)(n+4)}$[g(e_j ,Z)e_i - g(e_i ,Z)e_j +g(Fe_j ,Z)Fe_i–g(Fe_i ,Z)Fe_j -2g(Fe_i ,e_j)$\bar{Z}$]

Further putting Z= e_k in the above equation, we get

(2.08) $\widetilde{C}$(e_i ,e_j ,e_k) = K(e_i ,e_j, e_k) - $\frac{1}{(n+4)}$[g(Fe_j, e_k){λ_j Fe_i + λ_i Fe_i} - g(Fe_i, e_k){ λ_i Fe_j + λ_j Fe_j } -2g(Fe_i ,e_j){ λ_i Fe_k + λ_kFe_k }]
+ $\frac{k}{(n+2)(n+4)}$[g(Fe_j , e_k)Fe_i - g(Fe_i , e_k)Fe_j -2g(Fe_i ,e_j)Fe_k]

Contracting above equation with respect to e_i, we get

(2.09) C_1^1 $\widetilde{C}$(e_i ,e_j ,e_k) = C^*(e_j ,e_k) = 0, ∀ e_j ,e_k

So, we have,

Theorem (2.11): In a Cosymplectic manifold M_n admitting an orthonormal basis normal to the contact vector T, we have
C_1^1 $\widetilde{C}$(e_i ,e_j ,e_k) = C^*(e_j ,e_k) = 0, ∀ e_j ,e_k

Proof: The proof of the theorem immediately follows from the equation (2.09),

Again taking Z = e_j in (2.07), we get

(2.10) $\widetilde{C}$(e_i ,e_j ,e_j) = K(e_i ,e_j, e_j) - $\frac{1}{(n+4)}$[$\lambda_j e_i$ + $e_i\lambda_i$ + μT} - 3g(Fe_i , e_j){ λ_i Fe_j + λ_j Fe_j}]+ $\frac{k}{(n+2)(n+4)}$[e_i - 3g(Fe_i , e_j)Fe_j]

Or

(2.11)(a) '$\widetilde{C}$(e_i ,e_j ,e_j,e_i) = 'K(e_i ,e_j, e_j ,e_i) - $\frac{1}{(n+4)}$[(λ_i + λ_j) + 3g(Fe_i , e_j)^2(λ_i + λ_j)]+ $\frac{k}{(n+2)(n+4)}$[1 - 3g(Fe_i , e_j)^2]

Or

(2.11)(b) '$\widetilde{C}$(e_i ,e_j ,e_j,e_i) = 'K(e_i ,e_j, e_j ,e_i) + $\frac{1}{(n+4)}$[(λ_i + λ_j) + 3(λ_i + λ_j)g(Fe_i , e_j)^2]+ $\frac{k}{(n+2)(n+4)}$[1 - 3g(Fe_i , e_j)^2]

And by using (1.42), (1.43); (2.11)(b) can be rewritten as,

(2.12) '$\widetilde{C}$(e_i ,e_j ,e_j,e_i) = k^* + $\frac{1}{(n+4)}$[(λ_i + λ_j) +3(λ_i + λ_j)g(Fe_i , e_j)^2]- $\frac{k}{(n+2)(n+4)}$[1 - 3g(Fe_i , e_j)^2]

Theorem (2.12): In a Cosymplectic manifold M_n, admitting an orthonormal basis, given above, the sectional curvature k^* of M_n in the plane of unit vectors (e_i ,e_j) is given as,

(2.13) k^* + $\frac{1}{(n+4)}$[(λ_i + λ_j) +3(λ_i + λ_j)g(Fe_i , e_j)^2]- $\frac{k}{(n+2)(n+4)}$[1 - 3g(Fe_i , e_j)^2] = 0

Provided that H- Conformal curvature tensor vanishes in M_n.

Proof: In the equation (2.12) , if $\widetilde{C}$ = 0 , then (2.12) immediately follows.

Corollary (2.12): In a Cosymplectic manifold M_n, with the orthonormal basis, under consideration and with vanishing H – Conformal curvature tensor, the sectional curvature k^* is given as,

(2.14)(a) $k^* = -\frac{2(\lambda i + \lambda j)}{(n+4)} - \frac{2k}{(n+2)(n+4)}$

Provided that Fe_i is parallel to e_j

(2.14)(b) $k^* = -\frac{(\lambda i + \lambda j)}{(n+4)} - \frac{k}{(n+2)(n+4)}$

Provided that Fe_i is perpendicular to e_j

The proof of the above corollary follows immediately from above conditions and equation (2.13).

2. Conclusion

1. If { e_i ,Fe_j }, i= 1,2,3……….2m; be an orthonormal basis normal to T in M_n ,and if M_n be a cosymplectic manifold then H-Conformal curvature tensor $\widetilde{C}$ equals the Riemann-Curvature tensor in M_n .
2. Let M_n be a cosymplectic manifold admitting an orthonormal basis {e_i ,Fe_j }, i= 1,2,3……….2m; normal to T . Then H-Conformal curvature tensor $\widetilde{C}$ vanishes. if M_n is flat with respect to this basis.
3. In a Cosymplectic manifold M_n admitting an orthonormal basis normal to the contact vector T, we have $C_1^1 \widetilde{C}(e_i ,e_j ,e_k) = C^*(e_j ,e_k) = 0, \forall e_j ,e_k$
4. In a Cosymplectic manifold M_n, admitting an orthonormal basis, given above, the sectional curvature k^* of M_n in the plane of unit vectors (e_i,e_j) is given as,

$$k^* + \frac{1}{(n+4)}[(\lambda_i + \lambda_j) +3(\lambda_i + \lambda_j)g(Fe_i , e_j)^2] - \frac{k}{(n+2)(n+4)}[1 - 3g(Fe_i , e_j)^2] = 0$$

Provided that H- Conformal curvature tensor vanishes in M_n.

5. In a Cosymplectic manifold M_n, with the orthonormal basis, under consideration and with vanishing H – Conformal curvature tensor, the sectional curvature k^* is given as

$k^* = - \frac{2(\lambda i + \lambda j)}{(n+4)} - \frac{2k}{(n+2)(n+4)}$, Provided that Fe_i is parallel to e_j.

$k^* = - \frac{(\lambda i + \lambda j)}{(n+4)} - \frac{k}{(n+2)(n+4)}$, Provided that Fe_i is perpendicular to e_j.

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