

Design and Development of Flat Belt Dimpled Pulley and its Analysis on Slip

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Abstract: Belt transmissions are often used in industry machinery to transmit power. Another field of application concerns the Front End Accessory Drive of Automotive engine; it recently asks for better performances in order to allow development of new technological innovations as starter-alternator integration in transmission. Belt slip on pulley causes the transmission error. This can be controlled to some extent using appropriate initial tension. The flat belt pulley surface is plain. In the present study, the effect of positive geometry square and round shape dimpled surface instead of plain is investigated. The remaining parameters except the contact surface of belt and pulley are same. The experimental set up developed is used to determine the belt slip and slip torque for varying load and speed conditions. Two different dimple geometries are considered for this investigation. The paper elaborates the application of aluminum material liners on the flat pulley in order to improve performance. This is relatively new method. Results as to percentage slip versus load and percentage slip versus torque is plotted. For Geometrical profile design of square & round shape positive geometry flat belt pulley we use 2-d cad, for 3-d modeling we use Unigraphics N-x & for analysis we are using Ansys work bench 14.5, to obtain comparative analysis of plain, square dimpled & round dimpled pulley ANOVA is used.

Keywords: Belt transmissions, Contact surface, Flat belt pulley, Transmission error, Slip, Aluminium liner, Square, Round indent geometry.

1. Introduction

A. Belt Drives Applications

Flexible machine elements belt drives are called flexible machine elements. Flexible machine elements are used for a large number of industrial applications like Used for Transportation of coal, mineral ores etc. in conveying systems. over a long distance, Used for transmission of power. This is mainly used for running of various industrial appliances using prime movers like electric motors, I.C. Engine etc. and replacement of rigid type power transmission system. A belt transmission system Flexible machine element can replace gear drive as it has got an inherent advantage that, it can absorb a good amount of shock and vibration. In comparison to other transmission systems, it can take care of some degree of

misalignment between driven and long distance power transmission and the driver machines. For the all above reasons flexible machine elements are widely used in industrial application. Although we have some other flexible drives like roller chain drives, rope drive etc. we will discuss only about belt drives. A belt is a loop of flexible material which is used to link two or more rotating shafts mechanically, most often parallel. Belts can be used as a source of motion, to transmit power efficiently or to track relative movement. Belts are looped over pulleys and may have a twist between pulleys, and the shafts need not be parallel. In a two pulley system, the belt can either drive the pulleys normally in one direction, or the belt can be crossed, so that the direction of the driven shaft is reversed.

As belt transmits power by friction contact between the belt and the driving and driven sheave, Belt drives for power transmission are classed as frictional drives. Power transmission belts are available in various types: flat belts, V-belts, multi-ribbed belts and synchronous belts

To get the best service from any particular belt application, important points are to select the correct belt for the job & making sure that the belt is installed correctly and used properly.

B. Belt Selection Consideration

Environmental conditions in which the belt will operate, like, exposure to oil and grease, range of operating temperatures, abrasive dust and various chemical conditions, sunlight, and other weather conditions. Other considerations include are-Type of drive required, Driver/Driven Revolutions per Minute (RPM), Horsepower requirements, Pulley diameters and center distance, Take-up allowances and take-up design, Space limitation for operation, Pulsating or shock load conditions. Due to their versatility high efficiency synthetic flat belts are currently being used in all areas of power transmission technology, excellent properties, and outstanding reliability.

2. Literature Review

Power Transmission through Timing Belt in Two Wheeler

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Motors, by Gurumurthy Veerapathiran, Prabu Dhanapall, Ranjithkumar Koumaravel, Padmanaban Narayanamoorthy, Vignesh Ravi. In this paper the author studies the effect of noise and friction on performance of the chain drive system for motor bikes. Experiment shows that chain transmission in chain drive system leads to poor performance, due to its noise and chain gets loose because of aging and sprockets wear due to chain friction. The proposed system consists of drive and driven pulley with timing belt transmission. Compared to conventional method, proposed method give improved tension in pulleys and belt by additional arrangement called as belt tensioner. This gives good overall performance of the system, and reduces noise, vibration and gives high transmission speed.

Modelling, System Identification and Control of a Belt Drive System, by Shenjin Zhu (McMaster University, Canada). This paper to develop a mathematical model of an experimental belt drive system through physical modelling and system identification. This model is then used for the design of an advanced robust discrete-time controller. An extensive literature review is provided in this paper, covering modelling and control of belt drive system as well as sliding mode control (SMC) theory. Physical modelling is carried out for an experimental system followed by system identification. Both the physical and the identified models are used to analyse and investigate the characteristics of the system. Different control approaches such as discrete-time proportional integral derivative (DPID) and discrete-time sliding mode control (DSMC) are designed and implemented. The results are compared and conclusions are drawn from both control approaches.

Coupled belt-pulley mechanics in serpentine belt drives by Lingyuan Kong. This paper studies belt vibration and slip which are primary concerns in the design of serpentine belt drives. Belt-pulley coupling is essential for the analysis. This work investigates issues to advance the understanding of belt-pulley mechanics. This model can explain the transverse span vibrations caused by crankshaft pulley fluctuations at low engine idle speeds where other coupling mechanisms do not. For the steady state analysis, a novel transformation of the governing equations to a standard ODE form for general-purpose BVP solvers leads to numerically exact steady solutions. The approach is straightforward, suited for different boundary conditions, and has accessible physical

V-Belt Transmission Mechanics with Two Same Pulleys, by Qing Bao Wei, Xin Chen, Da Yu Zheng. This paper focus on the steady mechanics of V-belt drives in the state of sliding friction between groove and belt. Pulley groove results in two-dimensional radial and tangential friction forces whose direction depends on relative speed between belt and the pulley along the contact arc. The belt model is analyzed with comparing method which is performed through acquiring equivalent coefficient of the friction, and replacing the coefficient of friction in the flat belt mechanical equilibrium equations for v-belt analysis as well. V-belt is made of special anisotropic material, its material and pulley groove structure make V-belt drive mechanics complicated. The elasticity limit, unseating effects and seating, radial compliance makes us

consider the macro features of drive and not to consider the material heterogeneity. Through the sample calculation it's proved that this method is very simple and valid for V-belt mechanical analysis.

An Approach to Find out the Stresses Induced in Flat Belt during Half Rotation of a Driving Pulley by Shrikant A. Thote, M.K. Sonpimple, G. D. Mehta. The paper work basically emphasis on the case pertaining to a belt drive, which is used for flour mill operation. The operation of a flour mill is explained as under. A high capacity flour mill demand power approximately 27 Hp for crushing the grain into required form of flour. A bag of grain wheat is poured into hopper. The wheat grain continuously inserted through cavity to crushing stone. Generally, this crushing stone gets power through the belt drive. Due to crushing action of stone the wheat grain then get converted into desired flour. The quality of flour is governed by the gap between stone crusher. During production if belt drive fails then it may turns to production loss. At present this break down in flour mill industries seems to be frequent. The paper finally throws an eye on how to reduce this frequent damage with its significance and effects.

Rotational Response and Slip Prediction of Serpentine Belt Drives Systems by Hwang, S. J., Perkins, N. C., Ulsoy, A. G., and Meckstroth, R. J. This paper studies belt vibration and slip which are primary concerns in the design of serpentine belt drives. Belt-pulley coupling is essential for the analysis. The belt-pulley coupling is studied through evolution of vibration modes. When belt-pulley coupling is strong, the dynamic behavior of the system is quite different from that of the string model where there is no such coupling is present.

Transient Belt Stresses During Starting and Stopping: Elastic Response Simulated by Finite Element Methods by L. K. Nordell and Z. P. Ciozda, USA. This article present an introduction to different modern analysis techniques which is used in determining the magnitude of the dynamic transient forces propagated in a conveyor belt during its starting and stopping phases. Transient forces can be generated which Impair the integrity of conveyor system. Prediction, control, and allowance for all these forces are necessary for a successful design. When the belt-pulley coupling is strong, the dynamic behavior of the system is quite different from that of the string model Prediction of the transient behaviors has been accomplished with the aid of a computer modeling tool.

A. Literature Review summary

The literature review of this project mainly deals with application, advantages & methods of power transmission using belt drives. The effect of friction & noise on the chain drive studied and based on observations system consist of driving and driven pulley with belt drive arrangement is proposed. The proposed system with additional arrangement gives good overall performance on system & reduces noise & vibration resulting high transmission speed. Flat belt drives are used in application for power transmission in numerous machines where in large center distances are maintained and it is expected that the constant velocity ratio be maintained. The flat belt drives are normally used for high speed power transmission between

shafts. Even though the required characteristic flat belt is to transmit high speed power, slip is the major issue. The findings of studied papers can give us brief idea about application of square or round indented liners instead of permanent modified pulleys so as to increase inter changeability, easy fit and easy replacement after wear out of liner, just as it is commonly used in case of clutch or brake liner.

3. Problem Statement

Belt drives are most widely used in industry for transmission of power from one shaft to another shaft over considerable distance. It provides flexibility in positioning of motor relative to load. Pulleys of different diameters allow the speed of driven equipment to be increased or decreased relative to motor speed. Properly designed belt power transmission systems offer us high efficiency. It consists of an endless belt which is wrapped tightly over two pulleys which are mounted on separate shafts. The flat belt drives are normally used for high speed power transmission between shafts. Even though the required characteristic flat belt is to transmit high speed power, slip is the major issue. Thus it is necessary to address this issue by application of suitable remedial methods.

A. Power Loss in belt transmission system

Synchronous belt drive generates resonance on the belt span between two pulleys. When the transverse natural frequency of the belt matches with meshing frequency of the belt tooth and pulley tooth resonance occur. The resonance effects on accuracy of transmission [1]. Belt tensions are needed to be as small as possible to increase the belt fatigue strength and prolong bearing life and power loss from belt slip is unacceptable. If the belt speed is too high and considers the flexibility of belt then inertial forces is taken into consideration which is added in equilibrium equation includes tangential and radial inertial forces [2]. For keeping flat belt centered a crown pulley is used but it tends to cause belt tension concentration so it reduces the belt life. Misalignment between two flat pulleys is called as "skew" [3]. The pulley grooves generates two dimensional radial and tangential forces whose undetermined directions are dependent on relative speed between belt and pulley along the contact arc. A new computational technique developed to find steady mechanics of v-belt drive. This system analyses the speed, torque loss and maximum tension ratio [4].

Lateral belt motion in belt drive is investigated which includes belt over arbitrary shape with angular misalignment. In this cylindrical and conical pulley is considered, which identifies mean normal stress in the belt, its stiffness and width of belt [5]. Serpentine belt drives are used in front end accessory drive of automobile engine. Belt transmission is always used for higher capacity. In this case two types of tensioners are used to maintain minimum tension which insures power transmission and minimum slip. Two tensioners are dry friction tensioner and hydraulic tensioner [6]. The surface with groove pattern and mesh pattern have higher co-efficient of friction than the flat surfaces, but the surfaces with dimpled pattern have lower coefficient of friction than the flat surfaces. It has been reported that dimples on the surface acted as fluid reservoir during

sliding and this reduced the maximum contact pressure.

B. Various Techniques Used to reduce losses

Various techniques for texturing surface have been developed such as milling using machine tool, shot blasting, etching process, and laser beam process. Applying texturing on the surfaces is one way to improve the tribological properties [7]. Adhesive friction theory considers the friction force to be the product of real area of contact between sliding surfaces and shear stress at the contact. These have been separately measured by running different belts over transparent pulleys and observing the contact [8]. Dynamic coefficient of friction at any point on the contact arc is determined from slip and tension measurement on an operating belt and solution is found by utilization of experimental slip measurement [9].

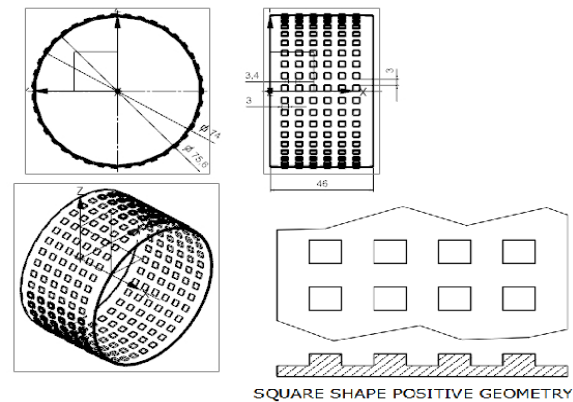


Fig. 1. Positive geometry square shape projection on dimpled pulley

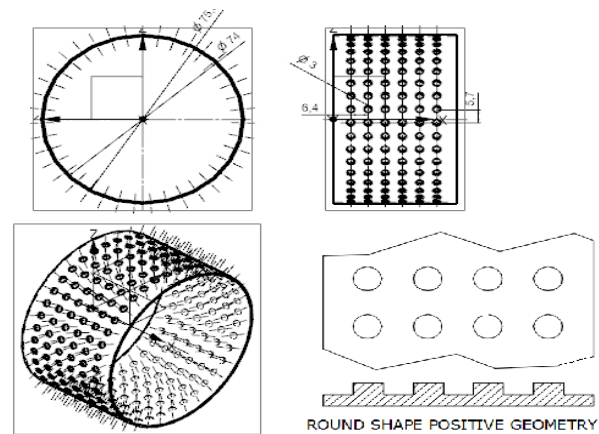


Fig. 2. Positive geometry round shape projection on dimpled pulley

C. Objective of project

1. Design development and analysis of square shape positive geometry flat belt pulley liner. Geometrical profile design using 2-d cad, 3-d modeling using Unigraphics N-x, analysis using Ansys work bench 14.5
2. Design development and analysis of round shape positive geometry flat belt pulley liner. Geometrical profile design using 2-d cad, 3-d modeling using Unigraphics N-x, analysis using Ansys work bench 14.
3. Test and trial to determine slip and characteristic of percentage slip Vs. load and percentage slip vs. torque.
4. Comparative analysis of plain, square dimpled and

round dimpled pulleys.

4. Design and Analysis

A. Design and Analysis of Flat belt pulley

1. 3D Model & Specifications: 3-D Modeling software Unigraphics N-x was used for generating model of flat belt pulley. Refer below indicated figures for flat belt pulley details.

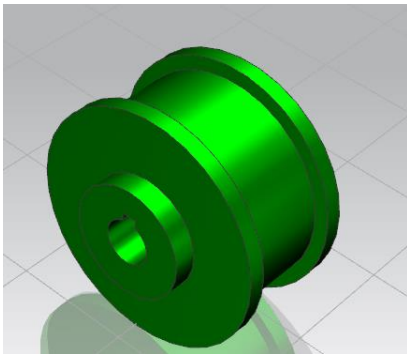


Fig. 3. 3-D Modeling of flat belt pulley in Unigraphics N-x

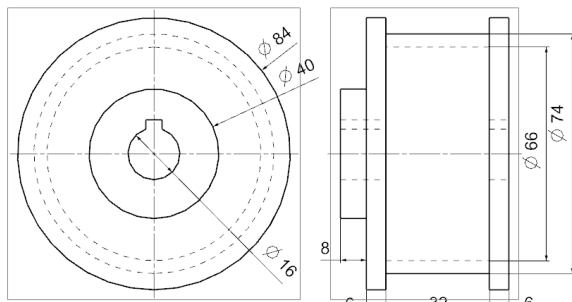


Fig. 4. 2-D Details of flat belt pulley

2. Specification:

Motor:

50 watt, 0 to 6000 rpm variable speed, 230 volt AC
 Gear details for drive from motor to pulley:
 Pinion on motor = 10 teeth 2 module
 Gear on driver shaft = 40 teeth = 2 module
 Driver shaft speed = $6000 / 5 = 1200$ rpm
 Torque @ driver shaft = 0.398 N-m = 0.4 N-m
 Considering Factor of safety = 2
 $T_{design} = 0 \times 0.4 = 0.8$ N-m

Material:

Designation	Ultimate Tensile strength N/mm ²	Yield strength N/mm ²
Aluminium	360	240

$\Rightarrow f_s \text{ max allowable} = 90$ N/mm² (considering factor of safety =4)

Check for torsional shear failure:

$$T = \frac{\pi}{16} \times f_s \text{ act} \times \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

$$800 = \frac{\pi}{16} \times f_s \text{ act} \times \left[\frac{74^4 - 66^4}{74} \right]$$

$$\Rightarrow f_s \text{ act} = 0.086017 \text{ N/mm}^2$$

As; $f_s \text{ act} < f_s \text{ all}$

\Rightarrow Pulley is safe under torsional load.

3. Meshing

Meshing is done on flat belt pulley for easy solving & accurate results in ANSYS 16.0. In 3D modelling only 2 types of meshes are possible namely hexa & tetra. In tetra meshing we have free meshing whereas for hexa meshing either of free or mapped meshing is available.

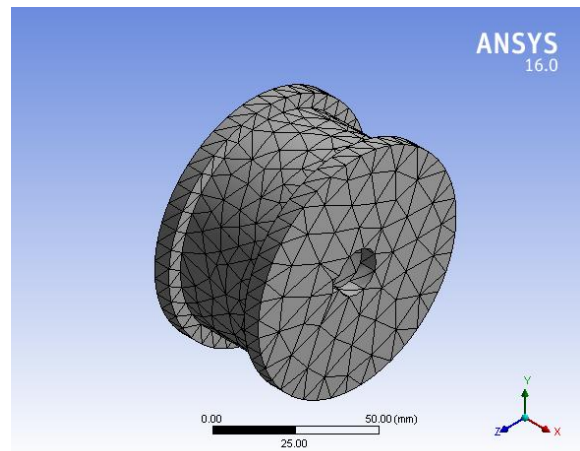


Fig. 5. Meshing of Flat belt pulley for structural analysis

4. Load and boundary conditions

Material assets play a main part in the result of the FE analysis. The material assets are single of the chief input to do FEA analysis & optimization. Applying boundary situation and exact loading is very essential to obtain accurate results of analysis.

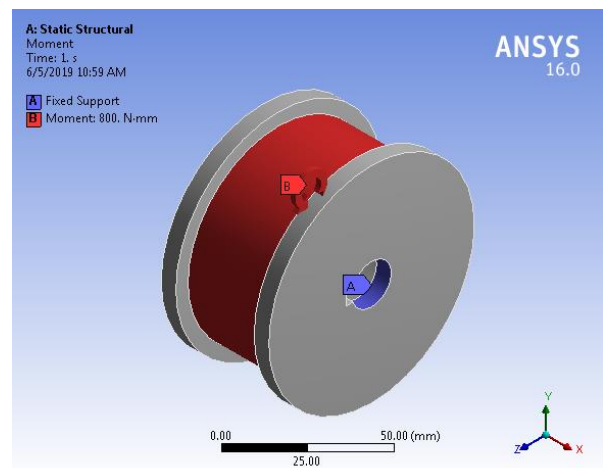


Fig. 6. Boundary condition applied on Flat belt pulley

5. Analysis of Flat belt pulley

We select aluminum as material for analysis next is the properties of substantial for structural analysis,

Material: Aluminum
 Poisson's ratio: 0.32
 Density: 2700 kg/m³

Examination of Flat belt pulley is completed by using properties of material given above. While analysis of Von-Mises Stresses, Deformation, Max principle stress are to be find out.

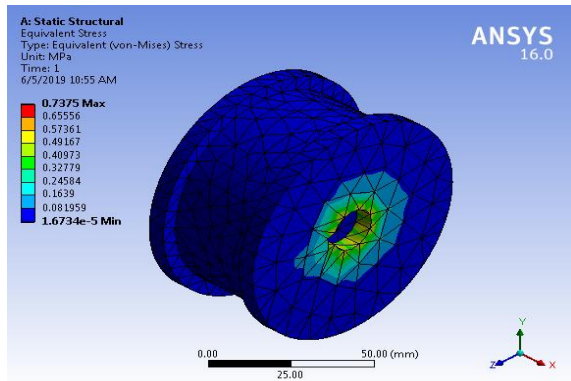


Fig. 7. Von Misses Stress distribution

As the maximum stress induced is 0.7375 Mpa which is less than the allowable stress hence the pulley is safe.

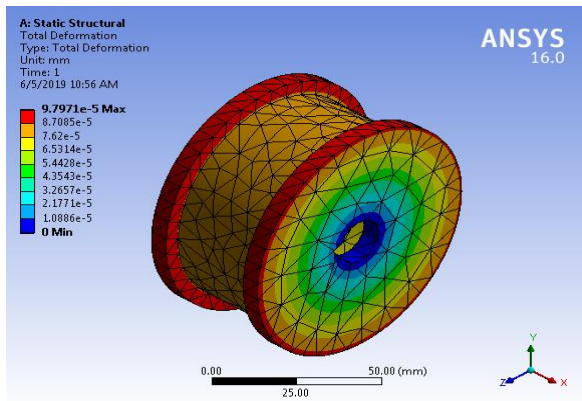


Fig. 8. Deformation of flat belt pulley

B. Design of Round indent geometry liner

1. 3D Model & Specifications: 3-D Modeling software Unigraphix N-x was used for generating model of round indent geometry liner. Refer below indicated figures for flat belt pulley details.

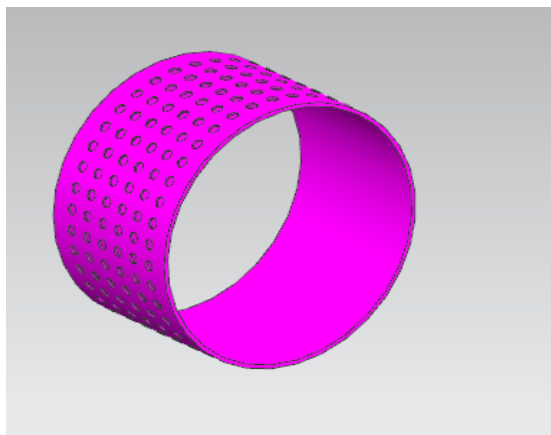


Fig. 9. 3-D Modeling round indent geometry liner in Unigraphix N-x

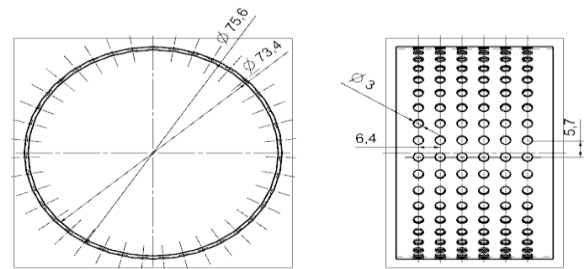


Fig. 10. 2-D Details of round indent geometry liner

2. Material selection

Designation	Ultimate Tensile strength N/mm ²	Yield strength N/mm ²
Aluminium	360	240

⇒ fs max allowable = 90 N/mm² (considering factor of safety =4)

Check for torsional shear failure:

$$T = \frac{\pi}{16} \times fs_{act} \times \left[\frac{Do^4 - Di^4}{74} \right]$$

$$800 = \frac{\pi}{16} \times fs_{act} \times \left[\frac{74^4 - 66^4}{74} \right]$$

⇒ fs_{act} = 0.26588N/mm²

As; fs_{act} < fs all

⇒ Round indent geometry liner is safe under torsional load.

3. Meshing

Meshing is done on round indent liner for easy solving & accurate results in ANSYS 16.0. In 3D modelling only 2 types of meshes are possible namely hexa & tetra. In tetra meshing we have free meshing whereas for hexa meshing either of free or mapped meshing is available.

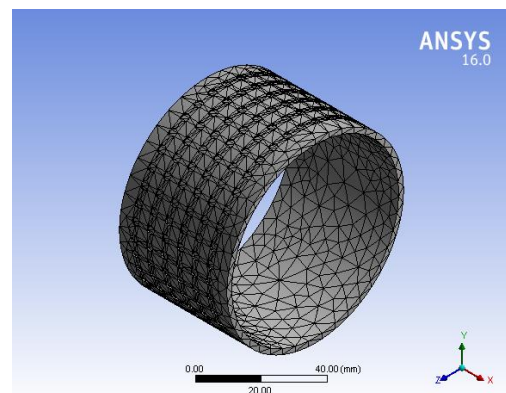


Fig. 11. Meshing of round indent liner for structural analysis

4. Load and boundary conditions

Material assets play a main part in the result of the FE analysis. The material assets are single of the chief input to do FEA analysis & optimization. Applying boundary situation and exact loading is very essential to obtain accurate results of analysis.

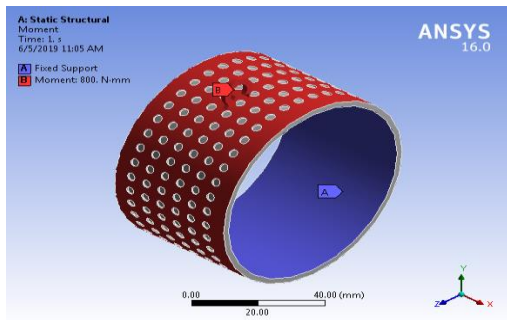


Fig. 12. Boundary condition applied on round indent liner

5. Analysis of Round indent liner

We select aluminum as material for analysis next is the properties of substantial for structural analysis,

Material: Aluminum

Poisson's ratio: 0.32

Density: 2700 kg/m³

Examination of round indent liner is completed by using properties of material given above. While analysis of Von-Mises Stresses, Deformation, Max principle stress are to be find out.

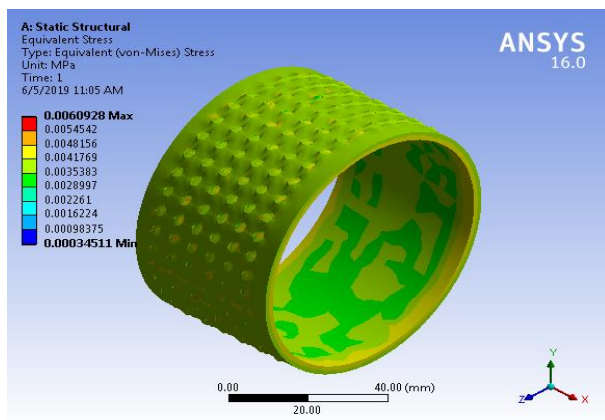


Fig. 13. Von Misses Stress distribution

As the maximum stress induced is 0.06 Mpa which is less than the allowable stress hence the pulley is safe.

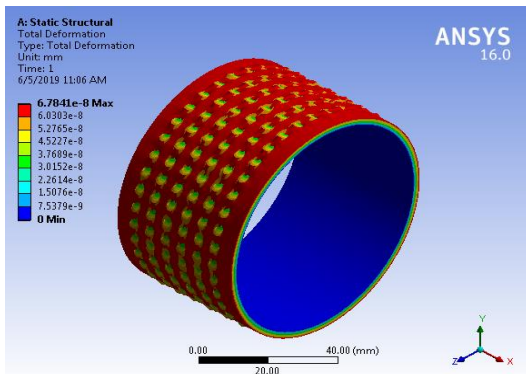


Fig. 14. Deformation of round indent liner

C. Design and Analysis of Square Geometry liner

- 3D Model & Specifications: 3-D Modeling software Unigraphics N-x was used for generating model of

square geometry liner. Refer below indicated figures for square geometry liner.

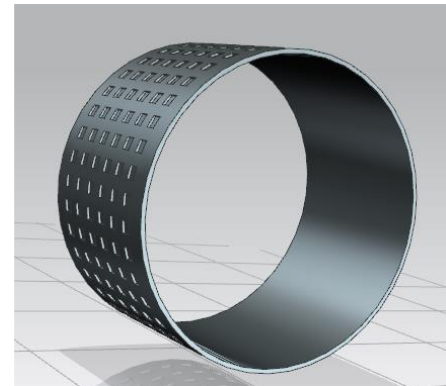


Fig. 15. 3-D Modeling of square geometry liner in Unigraphics N-x

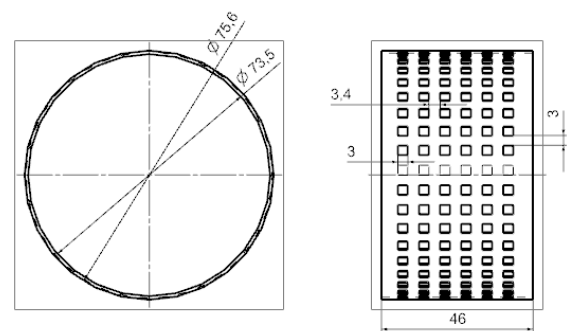


Fig. 16. 2-D Details of square geometry liner

2. Meshing

Meshing is done on square indent liner for easy solving & accurate results in ANSYS 16.0. In 3D modelling only 2 types of meshes are possible namely hexa & tetra. In tetra meshing we have free meshing whereas for hexa meshing either of free or mapped meshing is available.

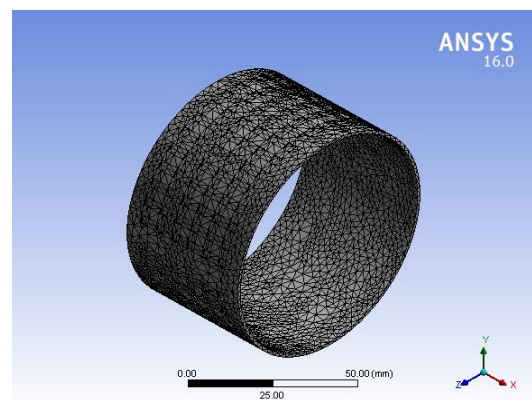


Fig. 17. Meshing of round indent liner for structural analysis

3. Load and boundary conditions

Material assets play a main part in the result of the FE analysis. The material assets are single of the chief input to do FEA analysis & optimization. Applying boundary situation and exact loading is very essential to obtain accurate results of analysis.

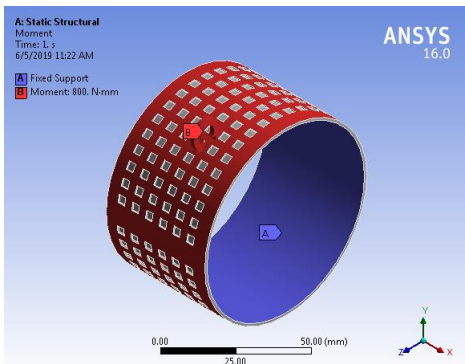


Fig. 18. Boundary condition applied on square indented liner

4. Analysis of round indented liner

We select aluminum as material for analysis next is the properties of substantial for structural analysis,

- Material: Aluminum
- Poisson's ratio: 0.32
- Density: 2700 kg/m³

Examination of round indented liner is completed by using properties of material given above. While analysis of Von-Mises Stresses, Deformation, Max principle stress are to be find out.

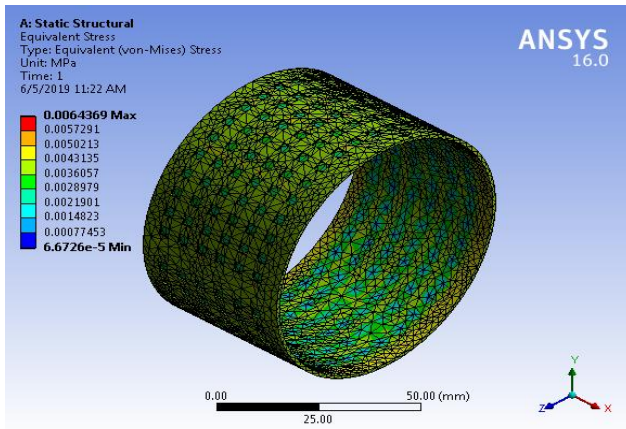


Fig. 19. Von Misses Stress distribution

As the maximum stress induced is 0.06 Mpa which is less than the allowable stress hence the pulley is safe.

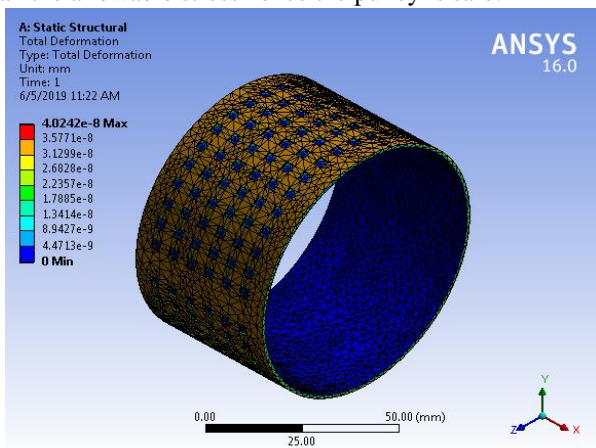


Fig. 20. Deformation of round indented liner

5. Experimental Setup

The selected mechanism and machine along with the damper will be designed with the help of following machines

- Centre lathe
- Milling machine
- DRO – Jig Boring machine
- Electrical Arc Welding

A. Experimental setup

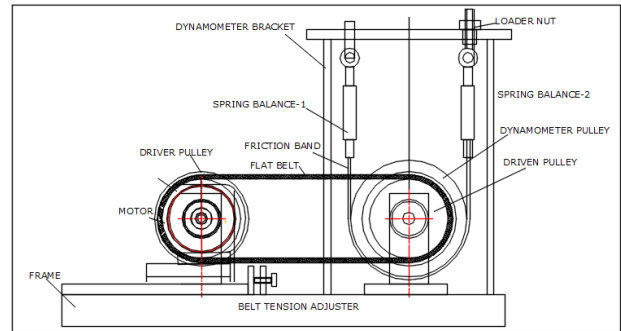


Fig. 21. Experimental Setup

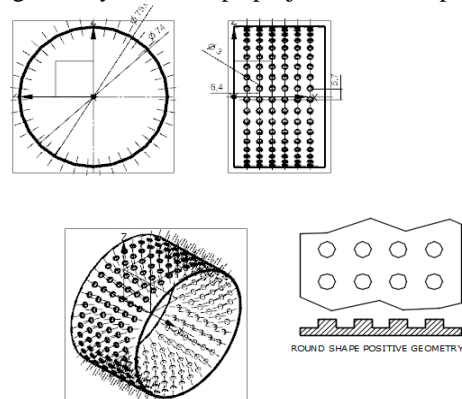
Set up is developed for measuring slip on driven pulley. A brake is applied on driven shaft with the help of load variation by nut and screw arrangement. Load is measured by spring balance attached to a brake drum. Set up includes driving pulley, driven pulley, flat belt (nylon) middle surface polyamide, Brake drum with friction band Motor specifications single phase ac variable speed with spur gear drive.

6. Procedure, Observations and Graphs

- To start motor turn electronic speed variator knob.
- Let mechanism run & stabilize at certain speed (say 2000 rpm)
- Note speed at no load condition
- Rotate the loader
- Tabulate the readings in the observation table
- Plot the following characteristic
 - Driven Pulley vs. Load
 - Torque vs. Load
 - Percentage Slip vs. Load

Case :1

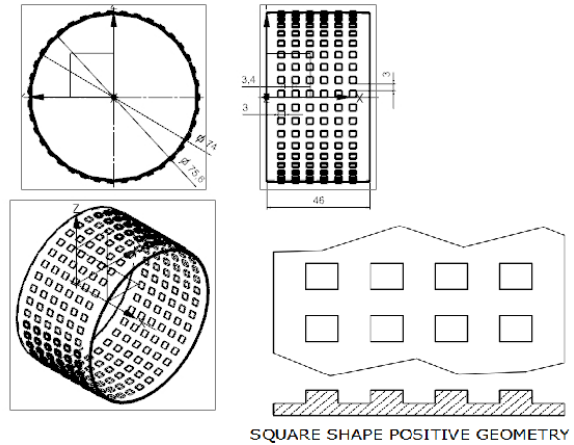
Positive geometry round shape projection on dimpled pulley:



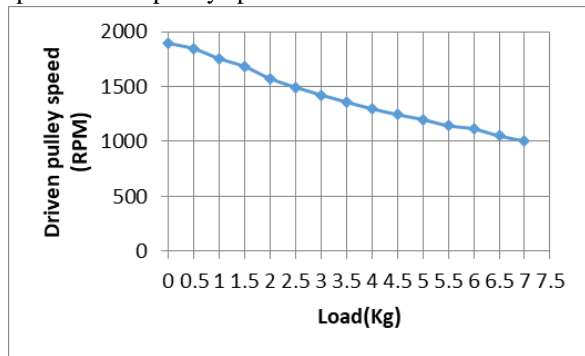
Observation table:

S. No.	Load (Kg)	Driver Pulley Speed (RPM)	Driven Pulley Speed (RPM)	Torque (N-m)	% Slip
1	0	1901	1893	0	0.420831
2	0.5	1856	1844	0.18639	0.646552
3	1	1774	1754	0.37278	1.127396
4	1.5	1701	1681	0.55917	1.175779
5	2	1592	1572	0.74556	1.256281
6	2.5	1520	1493	0.93195	1.776316
7	3	1456	1423	1.11834	2.266484
8	3.5	1392	1359	1.30473	2.37069
9	4	1333	1297	1.49112	2.700675
10	4.5	1283	1244	1.67751	3.039751
11	5	1239	1200	1.8639	3.1477
12	5.5	1190	1143	2.05029	3.94958
13	6	1161	1115	2.23668	3.962102
14	6.5	1096	1051	2.42307	4.105839
15	7	1050	1004	2.60946	4.380952

Case :2
Positive geometry square shape projection on dimpled pulley:



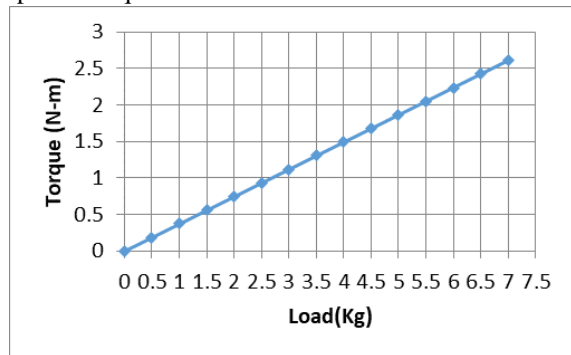
Graph of driven pulley speed vs. load:



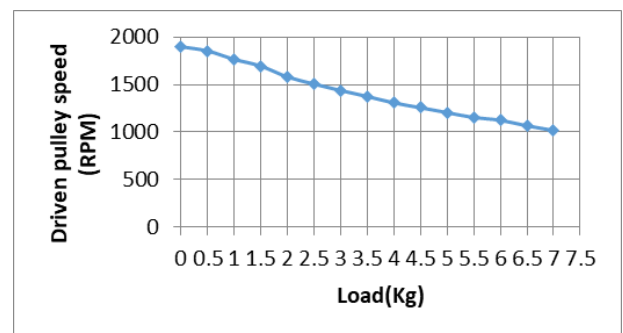
Observation Table:

S. No.	Load (Kg)	Driver Pulley Speed (RPM)	Driven Pulley Speed (RPM)	Torque (N-m)	% Slip
1	0	1900	1895	0	0.263158
2	0.5	1860	1852	0.18639	0.430108
3	1	1780	1764	0.37278	0.898876
4	1.5	1710	1694	0.55917	0.935673
5	2	1600	1580	0.74556	1.25
6	2.5	1530	1506	0.93195	1.568627
7	3	1465	1438	1.11834	1.843003
8	3.5	1405	1376	1.30473	2.064057
9	4	1340	1309	1.49112	2.313433
10	4.5	1290	1258	1.67751	2.48062
11	5	1245	1207	1.8639	3.052209
12	5.5	1200	1156	2.05029	3.666667
13	6	1170	1127	2.23668	3.675214
14	6.5	1110	1069	2.42307	3.693694
15	7	1060	1020	2.60946	3.773585

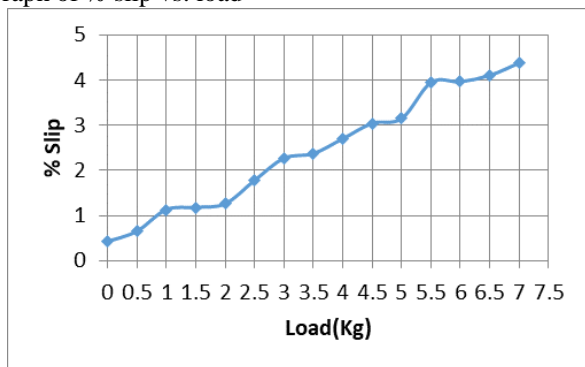
Graph of Torque vs. load



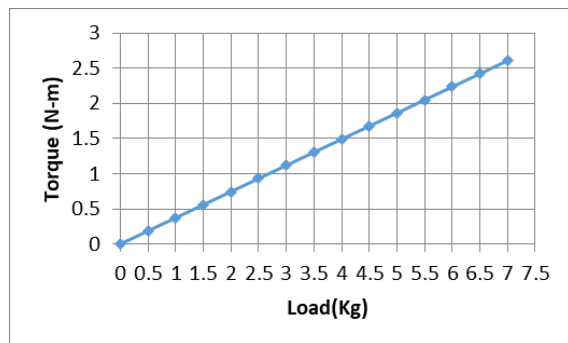
Graph of driven pulley speed vs. load:



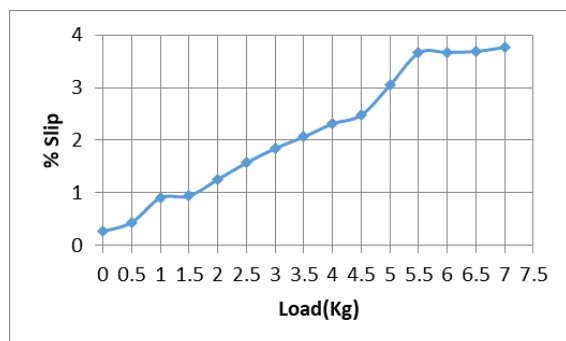
Graph of % slip vs. load



Graph of torque vs. load:



Graph of % slip vs. load:



7. Conclusion

Flat belt pulley surface is plain which causes more slip and affects efficiency. Effect of positive geometry square and round shape dimpled surface instead of plain is investigated.

Comparative analysis of plain, square dimpled and round dimpled pulleys, also test and trial to determine slip and characteristic of percentage slip vs. load and percentage slip vs. torque will give us information whether objectives of the project achieved or not.

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